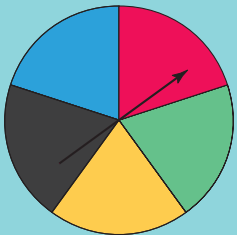


UNIT

7

Data Analysis

Many games involve probability. One game uses this spinner or a die labelled 1 to 6.



You can choose to spin the pointer or roll the die. You win if the pointer lands on red. You win if you roll a 6. Are you more likely to win if you spin the pointer or roll the die? Why do you think so?

What You'll Learn

- Find the mean, mode, median, and range of a set of data.
- Determine the effect of an outlier on the mean, median, and mode.
- Determine the most appropriate average to report findings.
- Express probabilities as ratios, fractions, and percents.
- Identify the sample space for an experiment involving two independent events.
- Compare theoretical and experimental probability.

Why It's Important

- You see data and their interpretations in the media. You need to understand how to interpret these data.
- You need to be able to make sense of comments in the media relating to probability.



Key Words

- mean
- mode
- measure of central tendency
- average
- range
- median
- outlier
- chance
- impossible event
- certain event
- independent events
- tree diagram
- sample space

7.1

Mean and Mode

Focus

Calculate the mean and mode for a set of data.

Questionnaires, experiments, databases, and the Internet are used to collect data. These collected data can be displayed in tables and graphs, which can be used to make predictions. In this lesson, you will learn ways to describe all the numbers in a data set.



Explore



You will need counters.
Three friends compared the time, in hours, they spent on the computer in one particular week.
Ali spent 5 h,
Bryn spent 9 h,
and Lynne spent 10 h.

Use counters to represent the time each person spent on the computer.
Find one number that best represents this time.







Reflect & Share

Share your findings with another pair of classmates.
How did you use counters to help you decide on the number?
Explain to your classmates why your number best represents the data.

Connect

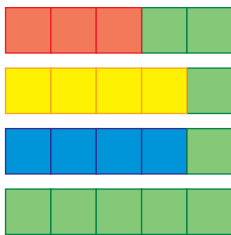
Allira surveyed 4 friends on the number of first cousins each has. To find a number that best represents the number of cousins, Allira used linking cubes.

Kinta		3
Orana		4
Illuka		4
Attunga		9



The **mean** is a number that can represent the centre of a set of numbers.

- One way to find the mean is to rearrange the cubes to make rows of equal length.



There are 5 cubes in each row.

The mean number of first cousins is 5.

When you make equal rows or columns, the total number of cubes does not change.

- You can use the total number of cubes to calculate the mean.
The number of cubes in each row is 3, 4, 4, and 9.
Add these numbers: $3 + 4 + 4 + 9 = 20$
Then divide by the number of rows, 4: $20 \div 4 = 5$
The mean is 5.

The **mode** is the number that occurs most often.

- To find the mode, determine which number occurs most often.
In Allira's data, the number 4 occurs twice.
The mode is 4 cousins.
Two people have 4 cousins.

In a set of data, there may be no mode or there may be more than one mode.

Each of the mean and the mode is a **measure of central tendency**. We say the word **average** to describe a measure of central tendency. An average is a number that represents all numbers in a set.

Example

Here are Ira's practice times, in seconds, for the 100-m backstroke:

121, 117, 123, 115, 117, 119, 117, 120, 122

Find the mean and mode of these data.

A Solution

To find the mean practice time, add the practice times:

$$121 + 117 + 123 + 115 + 117 + 119 + 117 + 120 + 122 = 1071$$

Divide by the number of data, 9: $1071 \div 9 = 119$

The mean practice time is 119 s.

The mode is the practice time that occurs most often.

117 occurs three times, so the mode practice time is 117 s.

Practice

- Use linking cubes to find the mean of each set of data.
 - 3, 4, 4, 5
 - 1, 7, 3, 3, 1
 - 2, 2, 6, 1, 3, 4
- Calculate the mean of each set of data.
 - 2, 4, 7, 4, 8, 9, 12, 4, 7, 3
 - 24, 34, 44, 31, 39, 32
- Find the mode of each set of data in question 2.
- Here are the weekly allowances for 10 Grade 7 students:
\$9, \$11, \$13, \$15, \$20, \$10, \$12, \$15, \$10, \$15
 - What is the mean allowance?
 - What is the mode allowance?
 - Suppose two allowances of \$19 and \$25 are added to the list. What is the new mean? What happens to the mode?
- Here are the ages of video renters at *Movies A Must* during one particular hour: 10, 26, 18, 34, 64, 18, 21, 32, 21, 54, 36, 16, 30, 18, 25, 69, 39, 24, 13, 22
 - What is the mean age? The mode age?
 - During another hour, the mode age of twelve video renters is 36. What might the ages of the renters be? Explain your answer.



6. Jordin Tootoo is the first Inuk athlete to play in the National Hockey League. On October 9, 2003, he played his first game for the Nashville Predators. This table shows Jordin's statistics when he played junior hockey for the Brandon Wheat Kings.

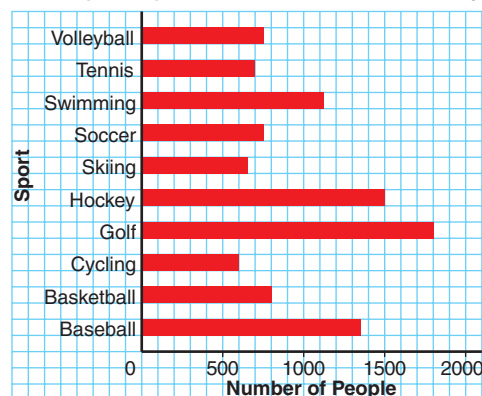
Jordin Tootoo's Scoring Records 1999-2003				
Year	Games Played	Goals	Assists	Points
1999-2000	45	6	10	16
2000-2001	60	20	28	48
2001-2002	64	32	39	71
2002-2003	51	35	39	74



Find the mean and mode for each set of data.

- Games Played
 - Goals
 - Assists
 - Points
7. **Assessment Focus** The graph shows the most popular sports of 13–15-year-olds in Wesley.
- Which sports are equally popular?
 - How could you use the bar graph to find the mode?
Explain and show your work.
 - Calculate the mean.
Use estimated values from the graph.

Most Popular Sports of 13-15-Year-Olds in Wesley



8. **Take It Further** A data set has 6 numbers. Four of the numbers are: 6, 3, 7, 9. Find the other two numbers in each case.
- The mean is 6.
 - The mode is 3 and the mean is 6.
Find as many different answers as you can.

Reflect

What is the difference between mean and mode?
Create a set of data to explain.

7.2

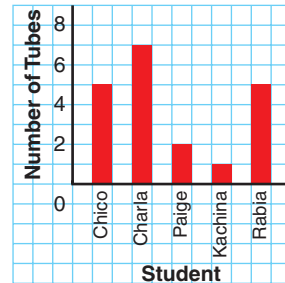
Median and Range

Focus Find the median and the range of a set of data.

The graph shows the number of tubes of hair gel used by each of 5 students in one particular month.

- How many tubes of gel did each student use?
- What is the mean number of tubes used?
- The mode number?
- How did you find the mean and the mode?

Number of Tubes of Hair Gel Used in One Particular Month



Explore



Your teacher will give you a bag of Cuisenaire rods.
You will need a ruler.

- Without looking, each person takes 3 rods from the bag.
Work together to arrange the 9 rods from shortest to longest.
Find the middle rod.
How many rods are to its right? To its left?
In what way is the middle rod typical of the rods your group picked?
What do you notice about the rods to the left and right of the middle rod?
- Each of you takes 1 more rod from the bag.
Place them among the ordered rods in the appropriate places.
Is there a middle rod now? Explain.
Sketch the rods.
Below each rod in your sketch, write its length.
How could you use the lengths to find a “middle” length?
How is the middle length typical of the rods in your sketch?



Reflect & Share

Is it possible to have two different sets of rods with the same middle length?
Share your results with other groups to find out.

Connect

The **median** of a data set is the middle number when the data are arranged in order.

- There are 11 Grade 7 students in Ms. Shim's combined Grades 6 and 7 class.
To find the median mark on the last science test, she listed their marks from greatest to least:

95, 92, 87, 85, 80, 78, 76, 73, 70, 66, 54

The middle number is 78.

There are 5 marks greater than 78, and 5 marks less than 78.

The median mark is 78.

- Another Grade 7 student transfers to Ms. Shim's class.
He writes the same test and receives a mark of 72.

To find the new median, the teacher includes his mark in the ordered list:

95, 92, 87, 85, 80, 78, 76, 73, 72, 70, 66, 54

There are two middle numbers, 78 and 76.

There are 5 marks greater than 78, and 5 marks less than 76.

The median is the mean of the 2 middle numbers:

$$(78 + 76) \div 2 = 77$$

The median mark is now 77.

- Now that the marks are arranged in order, we can easily find the range.
The **range** of a data set tells how spread out the data are.
It is the difference between the greatest and least numbers.
To find the range of the marks on the science test, subtract the least mark from the greatest mark:
 $95 - 54 = 41$
The range of the marks is 41.

When there is an odd number of data, the median is the middle number.

When there is an even number of data, the median is the mean of the two middle numbers.

The median is also a measure of central tendency, or an average.

When there is an even number of data, the median might *not* be one of the numbers in the data set.

Example

The hourly wages, in dollars, of 10 workers are: 8, 8, 8, 8, 9, 9, 9, 11, 12, 20

Find:

- a) the mean b) the mode c) the median d) the range

How does each average relate to the data?

A Solution

- a) Mean wage:

$$\text{Add: } 8 + 8 + 8 + 8 + 9 + 9 + 9 + 11 + 12 + 20 = 102$$

$$\text{Divide by the number of workers, 10: } 102 \div 10 = 10.2$$

The mean wage is \$10.20.

Three workers have a wage greater than the mean and 7 workers have a wage less than the mean.

- b) Mode wage:

8, 8, 8, 8, 9, 9, 9, 11, 12, 20

The mode wage is \$8. It occurs 4 times.

This is the least wage; that is, 6 workers have a wage greater than the mode.

- c) Median wage:

List the 10 wages in order from least to greatest:

8, 8, 8, 8, **9, 9, 9**, 11, 12, 20

The median wage is the mean of the 5th and 6th wages.

Both the 5th and 6th wages are 9.

The median wage is \$9.

There are 3 wages above the median and 4 wages below the median.

- d) Range:

8, 8, 8, 8, 9, 9, 9, 11, 12, 20

$$\text{Subtract the least wage from the greatest wage: } 20 - 8 = 12$$

The range of the wages is \$12.

Practice

- Find the median and the range of each set of data.
 - 85, 80, 100, 90, 85, 95, 90
 - 12 kg, 61 kg, 85 kg, 52 kg, 19 kg, 15 kg, 21 kg, 30 kg

- 2.** The Grade 7 students in two combined Grades 6 and 7 classes wrote the same quiz, marked out of 15.
Here are the results:
Class A: 8, 9, 9, 12, 12, 13, 13, 14, 15, 15
Class B: 10, 10, 11, 11, 12, 12, 13, 13, 14, 14
- Find the median mark for each class.
 - Find the range of each set of marks.
 - Which class do you think is doing better? Explain.
- 3.** a) Find the mean, median, and mode for each data set.
- | | |
|------------------------|----------------------------|
| i) 4, 5, 7, 8, 11 | ii) 50, 55, 65, 70, 70, 50 |
| iii) 7, 63, 71, 68, 71 | iv) 6, 13, 13, 13, 20 |
- b) Which data sets have:
- the same values for the mean and median?
What do you notice about the numbers in each set?
 - the same values for the mean, median, and mode?
What do you notice about the numbers in each set?
 - different values for the mean, median, and mode?
What do you notice about the numbers in each set?
- 4. Assessment Focus** Write two different data sets with 6 numbers, so that:
- The mode is 100. The median and the mean are equal.
 - The mode is 100. The mean is less than the median.
- Show your work.
- 5.** a) The median height of ten 12-year-old girls is 158 cm.
What might the heights be? How do you know?
- b) The mode height of ten 12-year-old boys is 163 cm.
What might the heights be? How do you know?
- 6.** Jamal was training for a 400-m race. His times, in seconds, for the first five races were: 120, 118, 138, 124, 118
- Find the median and mode times.
 - Jamal wants his median time after 6 races to be 121 s.
What time must he get in his 6th race? Show your work.
 - Suppose Jamal fell during one race and recorded a time of 210 s.
Which of the mean, median, and mode would be most affected? Explain.



7. In 2005, the Edmonton Miners hosted The Minto Cup Junior A Lacrosse Championship. Here are the 2005 statistics, as of June 30, 2005, for 10 players on the team.

Player	Games	Goals	Assists	Points	Penalty Minutes
Jeremy Boyd	13	2	8	10	54
Dan Claffey	11	3	11	14	33
Dalen Crouse	11	10	10	20	6
Andrew Dixon	15	4	5	10	47
Dan Hartzell	11	5	21	26	8
Cole Howell	12	21	13	34	0
Aiden Inglis	12	3	4	7	23
Ryan Polny	17	7	14	21	2
Chris Schmidt	5	8	4	12	2
Neil Tichkowsky	17	34	19	53	8

- a) Calculate the mean, the median, and the mode of each set of data.
- b) Make up a question about the mean, the median, or the mode that can be answered using these data. Answer your question.



8. **Take It Further** This is how Edward calculated the mean of these data.

48, 49, 50, 50, 51, 53, 57, 58

Estimated mean is 51.

Score	48	49	50	50	51	53	57	58
Deviation	-3	-2	-1	-1	0	+2	+6	+7

$$\begin{aligned} \text{Mean} &= 51 + \frac{(-3) + (-2) + (-1) + (-1) + 0 + 2 + 6 + 7}{8} \\ &= 52 \end{aligned}$$

Check that Edward's answer is correct. How does his method work?

Reflect

A median is the strip of land or concrete barrier separating lanes of highway traffic travelling in opposite directions. How is this meaning similar to its meaning in math?

7.3

The Effects of Outliers on Average

Focus Understand how mean, median, and mode are affected by outliers.

Explore



Students in a Grade 7 class measured their pulse rates.

Here are their results in beats per minute:

97, 69, 83, 66, 78, 8, 55, 82, 47, 52, 67, 76, 84,
64, 72, 80, 72, 70, 69, 80, 66, 60, 72, 88, 88

- Calculate the mean, median, and mode for these data.
- Are there any numbers that are significantly different from the rest? If so, remove them.

Calculate the mean, median, and mode again.

Explain how the three averages are affected.



Reflect & Share

Compare your results with those of another pair of classmates. How did you decide which numbers were significantly different? Why do you think they are so different?

Connect

A number in a set of data that is significantly different from the other numbers is called an **outlier**.

An outlier is much greater than or much less than most of the numbers in the data set.

Outliers sometimes occur as a result of error in measurement or recording. In these cases, outliers should be ignored.

Sometimes an outlier is an important piece of information that should not be ignored. For example, if one student does much better or much worse than the rest of the class on a test.

Outliers may not always be obvious. Identifying outliers is then a matter of choice.

Example

Here are the marks out of 100 on an English test for students in a Grade 7 class:
21, 23, 24, 24, 27, 29, 29, 29, 32, 37, 37, 38, 39,
40, 50, 50, 51, 54, 56, 57, 58, 59, 61, 71, 80, 99

- How many students were in the class? How do you know?
- What is the outlier? Explain your choice.
- Calculate the mean, median, and mode.
- Calculate the mean, median, and mode without the outlier.
What do you notice?
- Should the outlier be used when reporting the average test mark? Explain.

A Solution

- Count the number of marks to find the number of students in the class.
There are 26 students.
- There is only one number, 99, that is significantly different.
The outlier is 99.
The difference between the outlier and the nearest mark is $99 - 80 = 19$.
This difference is much greater than that between other pairs of adjacent marks.
- There are 26 marks. To find the mean mark, add the marks:
 $21 + 23 + 24 + 24 + 27 + 29 + 29 + 29 + 32 + 37 + 37 + 38 + 39 +$
 $40 + 50 + 50 + 51 + 54 + 56 + 57 + 58 + 59 + 61 + 71 + 80 + 99 = 1175$
Divide the total by the number of marks, 26: $1175 \div 26 \doteq 45.2$
The answer is written to the nearest tenth.
The mean mark is about 45.2.
The median mark is the mean of the 13th and 14th marks.
The 13th mark is 39. The 14th mark is 40.
So, the median is: $\frac{39 + 40}{2} = \frac{79}{2} = 39.5$
The mode is the mark that occurs most often. This is 29.
- Without the outlier, there are 25 marks and the sum of the marks is: $1175 - 99 = 1076$
The mean is: $1076 \div 25 = 43.04$
The median is the 13th mark: 39
The mode is 29.
When the outlier was removed, the mean and median decreased.
The mode remained the same.
- The outlier should be used when reporting the average test mark.
To understand how the class is performing, all test marks should be included.

Practice

- This set of data represents the waiting time, in minutes, at a fast-food restaurant:
5, 5, 5, 6, 5, 7, 0, 5, 1, 7, 7, 5, 6, 5, 5, 5, 8, 5, 0, 5, 4, 5, 2, 7, 9
 - Calculate the mean, median, and mode.
 - Identify the outliers. Explain your choice.
 - Calculate the mean, median, and mode without the outliers.
How is each average affected when the outliers are not included?
- Bryan recorded the time he spent on the school bus each day for one month. Here are the times, in minutes:
15, 21, 15, 15, 18, 19, 14, 20, 95, 18, 21, 14, 15, 20, 16, 14, 22, 21, 15, 19
 - Calculate the mean, median, and mode times.
 - Identify the outlier. How can you explain this time?
 - Calculate the mean, median, and mode times without the outlier.
How is each average affected when the outlier is not included?
 - A classmate asks Bryan, "What is the average time you spend on the bus each day?" How should Bryan answer? Give reasons.
- A clothing store carries pant sizes 28 to 46. A sales clerk records the sizes sold during her 6-h shift:
28, 36, 32, 32, 34, 4, 46, 44, 42, 38, 36, 36, 40, 32, 36
 - Calculate the mean, median, and mode sizes.
 - Is there an outlier? If so, why do you think it is an outlier?
 - Calculate the mean, median, and mode sizes without the outlier.
How is each average affected when the outlier is not included?
 - Should the outlier be used when the sales clerk reports the average pant size sold during her shift? Explain your thinking.
- Here are the science test marks out of 100 for the Grade 7 students in a combined-grades class:
0, 66, 65, 72, 78, 93, 82, 68, 64, 90, 65, 68
 - Calculate the mean, median, and mode marks.
 - Identify the outlier. How might you explain this mark?
 - Calculate the mean, median, and mode marks without the outlier.
How is each average affected when the outlier is not included?
 - Should the outlier be used when reporting the average test mark? Explain.

Remember to arrange the data in order before finding the median.



5. a) Give an example of a situation in which outliers would not be used in reporting the averages. Explain why they would not be included.
- b) Give an example of a situation in which outliers would be used in reporting the averages. Explain why they would be included.

6. **Assessment Focus** A Grade 7 class wanted to find out if a TV advertisement was true. The ad claimed that *Full of Raisins* cereal guaranteed an average of 23 raisins per cup of cereal. Each pair of students tested one box of cereal. Each box contained 20 cups of cereal. The number of raisins in each cup was counted.



- a) Assume the advertisement is true. How many raisins should there be in 1 box of cereal?
- b) Here are the results for the numbers of raisins in 15 boxes of cereal:
473, 485, 441, 437, 489, 471, 400, 453, 465, 413, 499, 428, 419, 477, 467
- Calculate the mean, median, and mode numbers of raisins.
 - Identify the outliers. Explain your choice.
 - Calculate the mean, median, and mode without the outliers. How do the outliers affect the mean?
 - Should the outliers be used when reporting the average number of raisins? Explain.
 - Was the advertisement true? Justify your answer.

7. **Take It Further** Here is a set of data: 2, 3, 5, 5, 7, 8
An outlier has been removed.

- a) Calculate the mean, median, and mode without the outlier.
- b) The outlier is returned to the set.
The averages become:
Mean: 7 Median: 5 Mode: 5
What is the outlier? Show your work.

Reflect

Your friend is having difficulty recognizing outliers in a data set. What advice would you give your friend?

Focus Understand which average best describes a set of data.

Explore

Record on the board how many siblings you have.
Use the class data.
Find the mean, the median, and the mode.
Find the range.

Reflect & Share

With a classmate, discuss which measure best describes the average number of siblings.



Connect

A clothing store sold jeans in these sizes in one day:

28 30 28 26 30 32 28 32 26 28 34 38 36 30 34 32 30

To calculate the mean jeans size sold, add the sizes, then divide by the number of jeans sold.

$$\begin{aligned}\text{Mean} &= \frac{28 + 30 + 28 + 26 + 30 + 32 + 28 + 32 + 26 + 28 + 34 + 38 + 36 + 30 + 34 + 32 + 30}{17} \\ &= \frac{522}{17} \\ &\doteq 30.7\end{aligned}$$

The mean size is approximately 30.7.

To calculate the median, order the jeans sold from least size to greatest size. There are 17 numbers.

The middle number is the median. The middle number is the 9th.

26, 26, 28, 28, 28, 28, 30, 30, **30**, 30, 32, 32, 32, 34, 34, 36, 38

The median size is 30.

The mode is the number that occurs most often.

But, there are two numbers that occur most often.

So, there are two modes.

They are 28 and 30.

So, the mode sizes are 28 and 30.



When there is an odd number of data, to find the middle number: Add 1 to the number of data, then divide by 2. This gives the position of the middle number. For example: $\frac{17 + 1}{2} = \frac{18}{2} = 9$; the middle number is the 9th.

In this situation, the mean, 30.7, is of little use.

The mean does not represent a size.

The median, 30, shows about one-half of the customers bought jeans of size 30 or smaller, and about one-half of the customers bought jeans of size 30 or larger.

The modes, 28 and 30, tell which sizes are purchased more often.

The mode is most useful to the storeowner.

He may use the mode to order extra stock of the most popular sizes.

Example

A bookstore has 15 books in its young adult section.

There are 5 different prices.

This table shows the number of books at each price.

- Find the mean, median, and mode prices.
- Which measure best represents the average price of a young adult book?
- What is the range of the prices?

Young Adult Books	
Price (\$)	Number of Books
8.99	3
9.99	5
13.99	5
32.99	1
37.99	1

A Solution

Make a list of the prices, in dollars:

8.99, 8.99, 8.99, 9.99, 9.99, 9.99, 9.99, 9.99, 13.99,
13.99, 13.99, 13.99, 13.99, 32.99, 37.99

a) Mean price:

- Multiply each price by the total number of books at that price, then add the prices.
 $(8.99 \times 3) + (9.99 \times 5) + (13.99 \times 5) + 32.99 + 37.99 = 217.85$
- Divide the total price by the total number of books: 15
 $\frac{217.85}{15} \div 14.52$, to two decimal places

The mean price is approximately \$14.52.

Median price:

There are 15 books.

The list shows the books in order from least price to greatest price.

The median price is the 8th price. The 8th price is \$9.99.

The median price is \$9.99.

Mode price:

There are two mode prices. They are \$9.99 and \$13.99.



- b) The mean price is not charged for any of the books.
Only two books cost more than the mean of \$14.52.

There are two mode prices.

One mode, \$9.99, is the same as the median price.

One-half the books cost the median price or less.

One-half cost more. So, the median price, \$9.99, best represents the average price of a young adult book at the store.

- c) For the range, subtract the lowest price from the highest price:

$$37.99 - 8.99 = 29.00$$

The range of prices is \$29.00.

The mean is usually the best average when no numbers in the data set are significantly different from the other numbers.

The median is usually the best average when there are numbers in the data set that are significantly different.

The mode is usually the best average when the data represent measures, such as shoe sizes or clothing sizes.

A store needs to restock the sizes that sell most often.

Practice

- The daily high temperatures for one week at Clearwater Harbour were: 27°C, 31°C, 23°C, 25°C, 28°C, 23°C, 28°C
 - Find the mean, median, and mode for these data.
 - Which average do you think best describes the daily high temperature at Clearwater Harbour that week? Explain.
 - The weather channel reported the average temperature for Clearwater Harbour that week was 23°C. Is this correct? Explain.
- Caitlin received these test marks in each subject.
 - Find the mean, median, and mode mark for each subject.
 - Explain what information each average gives.
 - Which subject do you think Caitlin is best at? Worst at? Explain your reasoning.



Caitlin's Marks

Math	85	69	92	55	68	75	78
Music	72	81	50	69	81	96	92
French	68	74	82	80	76	67	74

- 3.** The table shows the tips earned by five waiters and waitresses during two weeks in December.
- Calculate the mean, median, and mode tips for each week.
 - Calculate the mean, median, and mode tips for the two-week period.
 - Compare your answers in parts a and b. Which are the same? Which are different? Explain why.
 - Explain which average best represents the tips earned during the two weeks.

Weekly Tips Earned (\$)		
Waiter	Week 1	Week 2
James	1150	600
Kyrra	700	725
Tamara	800	775
Jacob	875	860
George	600	1165

- 4.** A small engineering company has an owner and 5 employees. This table shows their salaries.
- Calculate the mean, median, and mode annual salaries.
 - What is the range of the annual salaries?
 - Which measure would you use to describe the average annual salary in each case? Explain.
 - You want to attract a new employee.
 - You want to suggest the company does not pay its employees well.

Company Salaries	
Position	Annual Salary (\$)
Owner	130 000
Manager	90 000
2 Engineers	50 000
Receptionist	28 000
Secretary	28 000

- 5.** Is each conclusion correct? Explain your reasoning.
- The mean cost of a medium pizza is \$10.
So, the prices of three medium pizzas could be \$9, \$10, and \$11.
 - The number of raisins in each of 30 cookies was counted.
The mean number of raisins was 15.
So, in 10 cookies, there would be a total of 150 raisins.

- 6. Assessment Focus** In each case, which average do you think is most useful: the mean, median, or mode? Justify your answer.
- A storeowner wants to know which sweater sizes she should order.
Last week she sold 5 small, 15 medium, 6 large, and 2 X-large sweaters.
 - Five of Robbie's friends said their weekly allowances are: \$10, \$13, \$15, \$11, and \$10.
Robbie wants to convince his parents to increase his allowance.
 - Tina wants to know if her math mark was in the top half or bottom half of the class.

7. A quality control inspector randomly selects boxes of crackers from the production line. She measures their masses. On one day she selects 15 boxes, and records these data:
- 6 boxes: 405 g each
 - 4 boxes: 390 g each
 - 1 box: 380 g
 - 2 boxes: 395 g each
 - 2 boxes: 385 g each
- a) Calculate the mean, median, and mode masses.
 - b) What is the range of the masses?
 - c) For the shipment of crackers to be acceptable, the average mass must be at least 398 g. Which average would you use to describe this shipment to make it acceptable? Explain.



8. **Take It Further** Andrew has these marks: English 82%, French 75%, Art 78%, Science 80%
- a) What mark will Andrew need in math if he wants his mean mark in these 5 subjects to be each percent?
 - i) 80%
 - ii) 81%
 - iii) 82%
 - b) Is it possible for Andrew to get a mean mark of 84% or higher? Justify your answer.

9. **Take It Further** Celia received a mean mark of 80% in her first three exams. She then had 94% on her next exam. Celia stated that her overall mean mark was 87% because the mean of 80 and 94 is 87. Is Celia's reasoning correct? Explain.



Reflect

Use your answers from *Practice*. Describe a situation for each case.

- a) The mean is the best average.
 - b) The median is the best average.
 - c) The mode is the best average.
- Justify your choices.



Using Spreadsheets to Investigate Averages

Focus Investigate averages using a spreadsheet.

You can use spreadsheet software to find the mean, median, and mode of a set of data.

A spreadsheet program allows us to calculate the averages for large sets of data values quickly and efficiently.

You can also use the software to see how these averages are affected by outliers.

Here are the heights, in centimetres, of all Grade 7 students who were on the school track team: 164, 131, 172, 120, 175, 168, 146, 176, 175, 173, 155, 170, 172, 160, 168, 178, 174, 184, 189

In some spreadsheet software, the mean is referred to as the average.

Use spreadsheet software.

- Input the data into a column of the spreadsheet.
- Use the statistical functions of your software to find the mean, median, and mode. Use the Help menu if you have any difficulties.
- Investigate the effect of an outlier on the mean, median, and mode.

Delete 120.

What happens to the mean?

Median? Mode? Explain.

- Suppose one member of the track team with height 155 cm is replaced by a student with height 186 cm. How does this substitution affect the mean, median, and mode? Explain.

	A	B	C	D	E
1	164				
2	131				
3	172				
4	120				
5	175				
6	168				
7	146				
8	176				
9	175				
10	173				
11	155				
12	170				
13	172				
14	160				
15	168				
16	179				
17	174				
18	184				
19	189				
20	120				
21	172				
22	172				

	A	B	C
1	164		
2	131		
3	172		
4	120		
5	175		
6	168		
7	146		
8	176		
9	175		
10	173		
11	155		
12	170		

Notice that when you add or remove data values, the averages change to reflect the adjustments.

✓ Check

1. Enter these data into a spreadsheet.

They are the donations, in dollars, that were made to a Toy Wish Fund.

5, 2, 3, 9, 10, 5, 2, 8, 7, 15, 14, 17, 28, 30, 16, 19, 4, 7, 9, 11, 25, 30,
32, 15, 27, 18, 9, 10, 16, 22, 34, 19, 25, 18, 20, 17, 9, 10, 15, 35

a) Find the mean, median, and mode.

b) Add some outliers to your spreadsheet.

State the values you added.

How do the new mean, median, and mode compare to their original values? Explain.

2. Enter these data into a spreadsheet.

They are purchases, in dollars, made by customers at a grocery store.

55.40, 48.26, 28.31, 14.12, 88.90, 34.45, 51.02, 71.87, 105.12, 10.19,
74.44, 29.05, 43.56, 90.66, 23.00, 60.52, 43.17, 28.49, 67.03, 16.18,
76.05, 45.68, 22.76, 36.73, 39.92, 112.48, 81.21, 56.73, 47.19, 34.45

a) Find the mean, median, and mode.

b) Add some outliers to your spreadsheet.

State the values you added.

How do the new mean, median, and mode compare to their original values? Explain.



3. Enter these data into a spreadsheet.

They are the number of ice-cream bars sold at the community centre each day in the month of July.

101, 112, 127, 96, 132, 125, 116, 97, 124, 136, 123, 113, 78, 102, 118, 130,
87, 108, 114, 99, 126, 86, 94, 117, 121, 107, 122, 119, 111, 105, 93

Find the mean, median, and mode.

What happens when you try to find the mode? Explain.

4. Repeat question 1 parts a and b.

This time, enter data you find in the newspaper or on the Internet.

Reflect

List some advantages of using a spreadsheet to find the mean, median, and mode of a set of data. What disadvantages can you think of?

Mid-Unit Review

LESSON

- 7.1** **7.2** **1.** Here are the heights, in centimetres, of the students in a Grade 7 class:
162, 154, 166, 159, 170, 168, 158, 162, 172, 166, 157, 170, 171, 165, 162, 170, 153, 167, 164, 169, 167, 173, 170
- Find the mean, median, and mode heights.
 - What is the range of the heights?
- 2.** The mean of five numbers is 20. The median is 23. What might the numbers be? Find 2 different sets of data.
- 7.3** **3.** The cost of hotel rooms at *Stay in Comfort* range from \$49 to \$229 per night. Here are the rates charged, in dollars, for one particular night:
70, 75, 85, 65, 75, 90, 70, 75, 60, 80, 95, 85, 75, 20, 65, 229
- Calculate the mean, median, and mode costs.
 - Identify the outliers. How can you explain these costs?
 - Calculate the mean, median, and mode costs without the outliers. How is each average affected when the outliers are not included?
 - Should the outliers be used when reporting the average cost of a hotel room? Explain.
- 7.4** **4.** A quality control inspector measures the masses of boxes of raisins. He wants to know if the average mass of a box of raisins is 100 g. The inspector randomly chooses boxes of raisins. The masses, in grams, are:
99.1, 101.7, 99.8, 98.9, 100.8, 100.3, 98.3, 100.0, 97.8, 97.6, 98.5, 101.7, 100.2, 100.2, 99.4, 100.3, 98.8, 102.0, 100.3, 98.0, 99.4, 99.0, 98.1, 101.8, 99.8, 101.3, 100.5, 100.7, 98.7, 100.3, 99.3, 102.5
- Calculate the mean, median, and mode masses.
 - For the shipment to be approved, the average mass of a box of raisins must be at least 100 g. Which average could someone use to describe this shipment to get it approved? Explain.
- 5.** Is each conclusion true or false? Explain.
- The mode number of books read last month by students in James' class is 5. Therefore, most of the students read 5 books.
 - A random sample of 100 people had a mean income of \$35 000. Therefore, a random sample of 200 people would have a mean income of \$70 000.

Focus Express probabilities as ratios, fractions, and percents.

When the outcomes of an experiment are equally likely, the probability of an event occurring is:

$$\frac{\text{Number of outcomes favourable to that event}}{\text{Number of possible outcomes}}$$

Explore



At the pet store, Mei buys 100 biscuits for her dog, Ping-Ping. She buys 75 beef-flavoured biscuits, 15 cheese-flavoured, and 10 chicken-flavoured. The clerk puts them all in one bag. When she gets home, Mei shakes the bag and pulls out one biscuit.

- What is the probability that Mei pulls out a cheese-flavoured biscuit from the bag?
- How many different ways could you write this probability?
- What is the probability of pulling out a beef-flavoured biscuit? A chicken-flavoured biscuit? Write each probability 3 different ways.
- What is the probability of pulling out a vegetable-flavoured biscuit?
- What is the probability of pulling out a flavoured biscuit?



Reflect & Share

Compare your results with those of another pair of classmates. How many different ways did you write a probability? Are all of the ways equivalent? How do you know? What is the probability of an event that always occurs? An event that never occurs?

Connect

A probability can be written as a ratio, a fraction, and a percent.

Sam buys a box of different flavours of food for his cat.

In a box, there are 14 packets of fish flavour, 2 of chicken flavour, and 4 of beef flavour.

Sam takes a packet out of the box without looking.

What is the probability that he picks a packet of chicken-flavoured food?

There are 20 packets in a box of cat food.

- Using words:
Only 2 of the 20 packets are chicken.
So, picking chicken is unlikely.
- Using a fraction:
Two of the 20 packets are chicken.
The probability of picking chicken is $\frac{2}{20}$, or $\frac{1}{10}$.
- Using a ratio:
The probability of picking chicken is $\frac{1}{10}$.
We can write this as the part-to-whole ratio 1:10.
- Using a percent:
To express $\frac{2}{20}$ as a percent, find an equivalent fraction with denominator 100.

$$\frac{2}{20} = \frac{10}{100} \text{ or } 10\%$$

The diagram shows the fraction $\frac{2}{20}$ on the left and $\frac{10}{100}$ on the right, with an equals sign between them. Two curved arrows point from the denominator 20 to the denominator 100, each labeled with $\times 5$.

The chance of picking chicken is 10%.

When *all* the outcomes are favourable to an event, then the fraction:

$$\frac{\text{Number of outcomes favourable to that event}}{\text{Number of possible outcomes}}$$

has numerator equal to denominator, and the probability is 1, or 100%.

For example, the probability of picking a packet of cat food is: $\frac{20}{20} = 1$

When *no* outcomes are favourable to an event, then the fraction:

$$\frac{\text{Number of outcomes favourable to that event}}{\text{Number of possible outcomes}}$$

has numerator equal to 0, and the probability is 0, or 0%.

For example, the probability of picking a packet of pork-flavoured cat food is: $\frac{0}{20} = 0$

Recall that you can also use a calculator to help you write a fraction as a percent.

When we express a probability as a percent, we often use the word *chance* to describe it.



The probability that an **impossible event** will occur is 0, or 0%.

The probability that a **certain event** will occur is 1, or 100%.

All other probabilities lie between 0 and 1.

Example

Twenty-five cans of soup were immersed in water.

Their labels came off so the cans now look identical.

There are: 2 cans of chicken soup; 4 cans of celery soup;

5 cans of vegetable soup; 6 cans of mushroom soup;

and 8 cans of tomato soup.

One can is picked, then opened.

- a) What is the probability of each event?

Write each probability as a ratio, fraction, and percent.

- i) The can contains celery soup.
 - ii) The can contains fish.
 - iii) The can contains celery soup or chicken soup.
 - iv) The can contains soup.
- b) State which event in part a is:
- i) certain
 - ii) impossible



A Solution

- a) There are 25 cans, so there are 25 possible outcomes.

- i) Four cans contain celery soup.

The probability of opening a can of celery soup is:

$$4:25, \text{ or } \frac{4}{25} = \frac{16}{100}, \text{ or } 16\%$$

- ii) None of the cans contain fish.

The probability of opening a can of fish is: 0, or 0%

- iii) Four cans contain celery soup and two contain chicken soup.

This is 6 cans in all.

The probability of opening a can of celery soup or chicken soup is:

$$6:25, \text{ or } \frac{6}{25} = \frac{24}{100}, \text{ or } 24\%$$

- iv) Since all the cans contain soup, the probability of opening a can of soup is:

$$20:20, \text{ or } \frac{20}{20}, \text{ or } 100\%$$

- b) i) The event that is certain to occur is opening a can that contains soup.

This event has the greatest probability, 1.

- ii) The event that is impossible is opening a can that contains fish.

This event has the least probability, 0.

Practice

Use a calculator when you need to.

1. Write the probability of each event as many different ways as you can.
 - a) Roll a 3 or 5 on a die labelled 1 to 6.
 - b) January immediately follows June.
 - c) Pick an orange out of a basket that contains 2 oranges, 6 apples, and 8 peaches.
 - d) The sun will set tomorrow.

2. A bag contains these granola bars: 12 apple, 14 peanut butter, 18 raisin, and 10 oatmeal. You pick one bar at random.

Find the probability of picking:

- a) a peanut butter granola bar
- b) an apple granola bar

3. Two hundred fifty tickets for a draw were sold.

One ticket, drawn at random, wins the prize.

- a) Joe purchased 1 ticket.

What is the probability Joe will win?

- b) Maria purchased 10 tickets.

What is the probability Maria will win?

- c) Ivan purchased 25 tickets.

What is the probability Ivan will *not* win?

Express each probability three ways.

4. Thanh has 20 felt pens in a pencil case.

He has 6 blue pens, 5 red pens, 2 yellow pens, 3 green pens,

2 brown pens, 1 purple pen, and 1 orange pen.

Thanh reaches into the case without looking and pulls out one pen.

Write a ratio, fraction, and percent to describe

the probability that Thanh picks:

- a) either a yellow or a green pen
- b) either a blue or a red pen
- c) a coloured pen
- d) a grey pen
- e) a purple pen



5. The names of 8 students are in a hat. You pick one name without looking. Find each probability. Express each probability as many ways as you can.



- Laura will be picked.
- Jorge will *not* be picked.
- A three-letter name will be picked.
- A five-letter name will be picked.
- A name with 4 or more vowels will be picked.
- A boy's or a girl's name will be picked.

6. Think of an experiment for which an event occurs with each probability. Explain your choice.

- a) 100% b) $\frac{1}{2}$ c) 1:6 d) 0

7. **Assessment Focus** Construct a spinner with red, yellow, blue, and green sectors, so the following probabilities are true.

- The probability of landing on red is $\frac{1}{5}$.
- The probability of landing on yellow is 50%.
- The probability of landing on blue is 1:10.
- The probability of landing on green is $\frac{2}{10}$.

Explain how you drew your spinner.

8. **Take It Further** A box contains 3 red, 2 green, and 4 white candies.

Carmen picked one candy, found it was white, and ate it.

She picked a second candy at random, found it was red, and ate it.

Carmen picked a third candy at random.

- Which colour is the third candy most likely to be? Explain.
- Write the probability that the third candy will be the colour named in part a. Use a ratio, fraction, and percent to write the probability.
- What is the probability that the candy will *not* be the colour named in part a?

Reflect

The weather forecast shows a 90% chance of rain tomorrow. How would this affect your plans for a class picnic? Why?

7.6

Tree Diagrams

Focus Investigate outcomes of probability experiments.

Recall that an outcome is the possible result of an experiment or action.
When you roll a die, the outcomes are equally likely.
When you toss a coin, the outcomes are equally likely.
Some experiments have two or more actions.

Explore



You will need a die labelled 1 to 6 and a coin.

- List the possible outcomes of rolling the die and tossing the coin.
How many possible outcomes are there?
How many outcomes include rolling a 4? Tossing a head?
- What is the theoretical probability of the event “a head on the coin and a 2 on the die?”
- Conduct the experiment.
One of you tosses the coin and one rolls the die.
Record the results.
Calculate the experimental probability of the event “a head on the coin and a 2 on the die” after each number of trials.
 - 10 trials
 - 20 trials
 - 50 trials
 - 100 trials
- How do the experimental and theoretical probabilities compare?



Reflect & Share

Compare the strategy you used to find the outcomes with that of another pair of classmates. Was one strategy more efficient than another? Explain.
Compare your probabilities. Combine your results to get 200 trials.
What is the experimental probability of the event “a head on the coin and a 2 on the die?”
How do the experimental and theoretical probabilities compare?

Connect

Two events are **independent events** if the result of the one event does not depend on the result of the other event.

Tossing two coins is an example of two independent events.

The outcome of the first toss does not affect the outcome of the second toss.

The outcome of the second toss does not depend on the outcome of the first toss.

We can use a **tree diagram** to show the possible outcomes for an experiment that has two independent events.

When 2 coins are tossed, the outcomes for each coin are heads (H) or tails (T).

List the outcomes of the first coin toss.

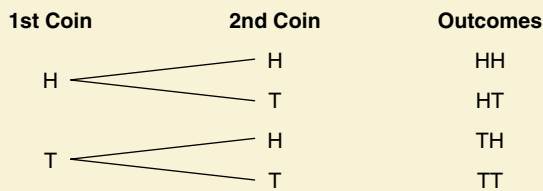
This is the first branch of the tree diagram.

For each outcome, list the outcomes of the second coin toss.

This is the second branch of the tree diagram.

Then list the outcomes for the coins tossed together.

You could also use a table to list the outcomes.



		First Coin	
		H	T
Second Coin	H	HH	HT
	T	TH	TT

There are 4 possible outcomes: HH, HT, TH, TT

Example

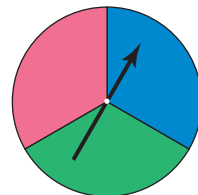
On this spinner, the pointer is spun once.

The colour is recorded.

The pointer is spun a second time.

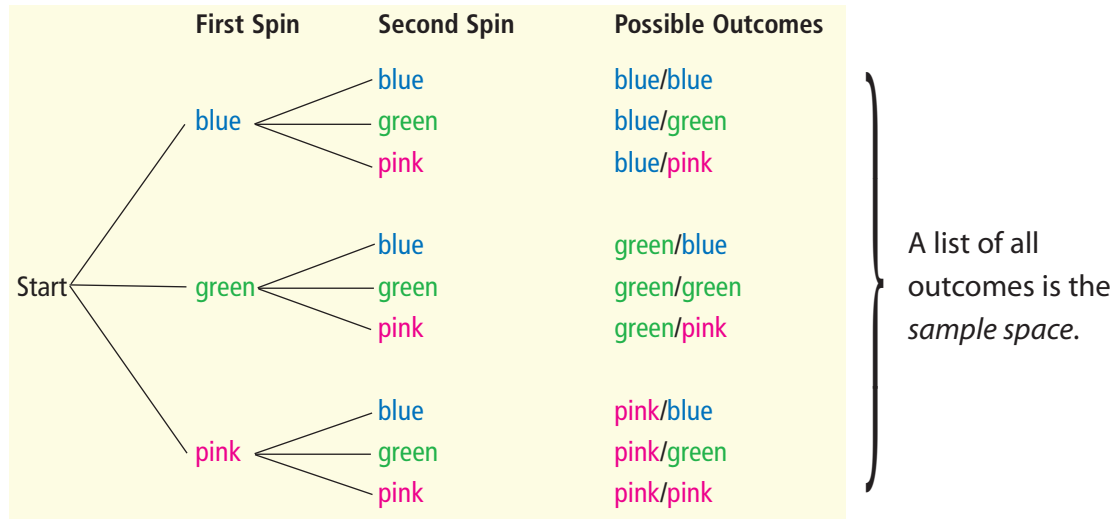
The colour is recorded.

- Draw a tree diagram to list the possible outcomes.
- Find the probability of getting the same colours.
- Find the probability of getting different colours.
- Carina and Paolo carry out the experiment 100 times. There were 41 same colours and 59 different colours. How do the experimental probabilities compare to the theoretical probabilities? Explain.



A Solution

- a) The first branch of the tree diagram lists the equally likely outcomes of the first spin: blue, green, pink
The second branch lists the equally likely outcomes of the second spin: blue, green, pink
For each outcome from the first spin, there are 3 possible outcomes for the second spin.
Follow the paths from left to right. List all the possible outcomes.



- b) From the tree diagram, there are 9 possible outcomes.
3 outcomes have the same colours: blue/blue, green/green, pink/pink
The probability of the same colour is:
 $\frac{3}{9} = \frac{1}{3} \doteq 0.33$, or about 33%
- c) 6 outcomes have different colours: blue/green, blue/pink, green/blue, green/pink, pink/blue, pink/green
The probability of different colours is:
 $\frac{6}{9} = \frac{2}{3} \doteq 0.67$, or about 67%
- d) The experimental probability of the same colour is:
 $\frac{41}{100} = 0.41$, or 41%
The experimental probability of different colours is:
 $\frac{59}{100} = 0.59$, or 59%
These probabilities are different from the theoretical probabilities.
The greater the number of times the experiment is carried out, the closer the theoretical and experimental probabilities may be.

Practice

1. List the sample space for each pair of independent events.

Why are the events independent?

- a) Rolling a die labelled 3 to 8 and tossing a coin



- b) Rolling a tetrahedron labelled 1 to 4 and spinning a pointer on this spinner

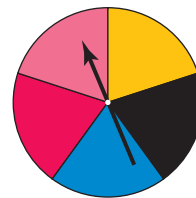
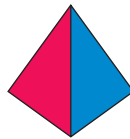


- c) Rolling a pair of dice labelled 1 to 6



2. Use the outcomes from question 1a. Aseer wins if an odd number or a head shows. Roberto wins if a number less than 5 shows. Who is more likely to win? Explain.
3. Use the outcomes from question 1b. Name an outcome that occurs about one-half of the time.
4. Use the outcomes from question 1c. How often are both numbers rolled greater than 4?
5. Hyo Jin is buying a new mountain bike. She can choose from 5 paint colours—black, blue, red, silver, or gold—and 2 seat colours—grey or black.
- a) Use a table to display all the possible combinations of paint and seat colours.
- b) Suppose Hyo Jin were to choose colours by pointing at lists without looking. What is the probability she would end up with a silver or black bike with a grey seat?

6. Assessment Focus Tara designs the game *Mean Green Machine*. A regular tetrahedron has its 4 faces coloured red, pink, blue, and yellow. A spinner has the colours shown. When the tetrahedron is rolled, the colour on its face down is recorded.



A player can choose to:

- roll the tetrahedron and spin the pointer, or
 - roll the tetrahedron twice, or
 - spin the pointer twice
- To win, a player must make green by getting blue and yellow. With which strategy is the player most likely to win? Justify your answer. Play the game to check. Show your work.



7. An experiment is: rolling a regular octahedron, labelled 1 to 8, and drawing a counter from a bag that contains 4 counters: green, red, yellow, blue

The number on the octahedron and the colour of the counter are recorded.

a) Use a regular octahedron, and a bag that contains the counters listed above.

Carry out the experiment 10 times. Record your results.

b) Combine your results with those of 9 classmates.

c) What is the experimental probability of each event?

- i) green and a 4
- ii) green or red and a 7
- iii) green or yellow and an odd number

d) Draw a tree diagram to list the possible outcomes.

e) What is the theoretical probability of each event in part c?

f) Compare the theoretical and experimental probabilities of the events in part c.

What do you think might happen if you carried out this experiment 1000 times? Explain.

Reflect

Which method do you prefer to find the sample space? Why?

Math Link

Your World

The Canadian Cancer Society runs a lottery every year to raise money for cancer research in Canada. One year, the chances of winning were given as the ratio 1:12. This could also be represented by the fraction $\frac{1}{12}$.



All the Sticks

This game is based on a game originally played by the Blackfoot Nation. The original materials were 4 animal bones and sticks.

HOW TO PLAY THE GAME:

1. Decorate:
 - 2 popsicle sticks with a zigzag pattern on one side
 - 1 popsicle stick with a circle pattern on one side
 - 1 popsicle stick with a pattern of triangles on one side
 Leave the other side of each popsicle stick blank.
2. Decide who will go first.
3. Place the counters in a pile on the floor.
4. Hold the 4 popsicle sticks in one hand, then drop them to the floor. Points are awarded according to the patterns that land face up. Find your pattern in the chart to determine your score. Take that number of counters. For example, if you score 4 points, take 4 counters. Take the counters from the pile until it has gone, then take counters from each other.

YOU WILL NEED

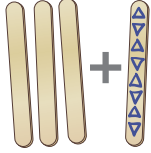
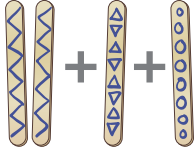
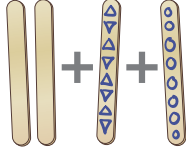
4 popsicle sticks (or tongue depressors); markers;
12 counters

NUMBER OF PLAYERS

2

GOAL OF THE GAME

To get all 12 counters

Pattern				Any other combination
Points	6	4	2	0

What is the theoretical probability of scoring 6 points?
How many points are you most likely to score in one turn?
How did you find out?

5. Take turns.
The first player to have all 12 counters wins.

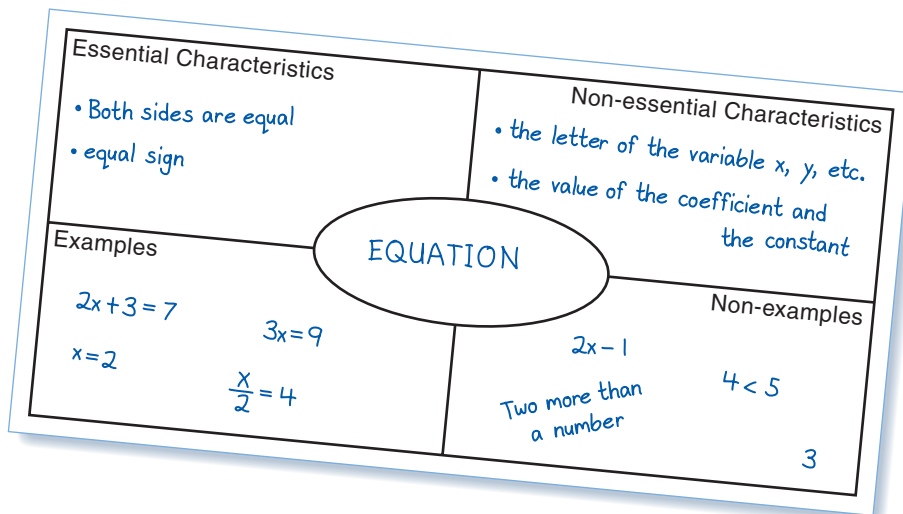
Using a Frayer Model



Every new math topic has new words and ideas.

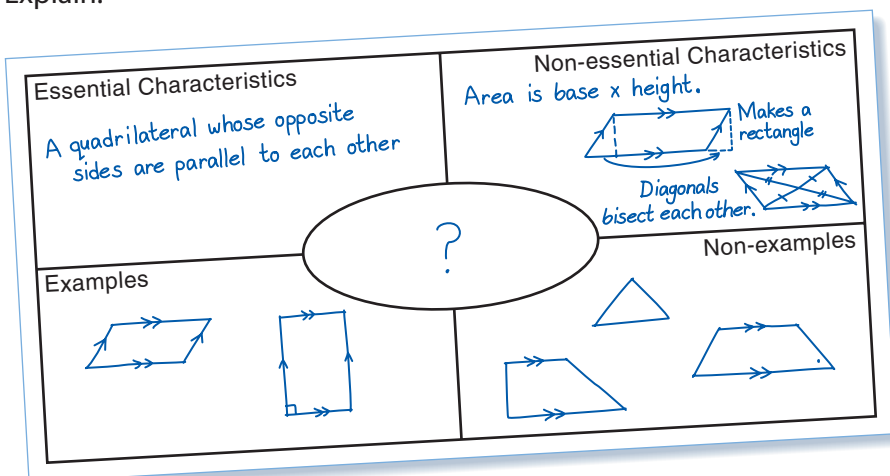
You can use a **Frayer Model** to help you remember new words and to better understand new ideas. A Frayer Model helps you make connections to what you already know.

Here is an example of a Frayer Model.



This model lists the essential and non-essential characteristics of the word.

What word do you think belongs in the centre of this Frayer Model? Explain.



✓ Check

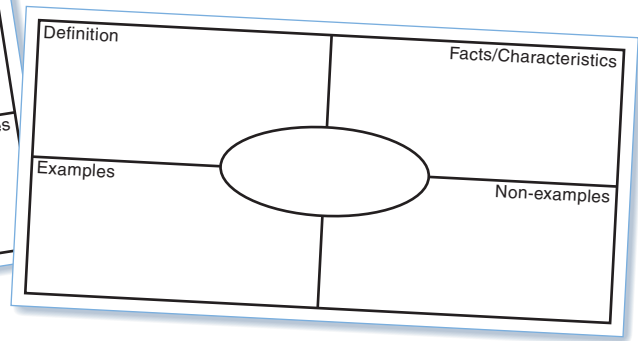
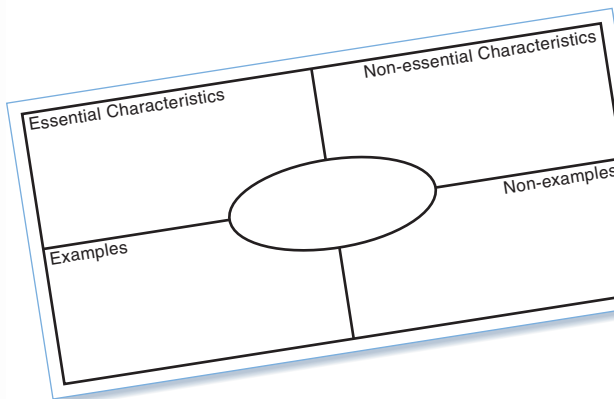
Work with a partner.

1. With your partner, make a Frayer Model for 3 of these words.

- Mean
- Median
- Mode
- Range
- Outliers
- Independent events
- Probability
- Tree diagram



Choose the Frayer Model type that best suits each word.



2. Work on your own.

Choose a word you did not use in question 1. Each of you should choose a different word. Make a Frayer Model.

This model gives the definition of the word, then lists its characteristics.

3. Share your Frayer Model with your partner.

- Suggest ways your partner could improve his Frayer Model.
- Make changes to reflect your partner's suggestions.

4. Discuss with your partner.

- Did making Frayer Models help you understand the new words and ideas? Explain.
- Do you think your Frayer Models will be a useful review tool? Explain.



Unit Review

What Do I Need to Know?

- ✓ Each of the *mean*, *median*, and *mode* is a measure of central tendency, or an average.
- ✓ In a set of data:
 - The *mean* is the sum of the numbers divided by the number of numbers in the set.
 - The *median* is the middle number when the data are arranged in order. When there is an odd number of data in the set, the median is the middle number. When there is an even number of data, the median is the mean of the two middle numbers.
 - The *mode* is the number that occurs most often.
 - A set of data can have no mode, one mode, or more than one mode.
 - The *range* is the difference between the greatest and least numbers.
 - An *outlier* is a number that is much greater than or much less than most of the numbers in the data set.
- ✓ Probability Range:
 - The probability of an event that is impossible is 0, or 0%.
 - The probability of an event that is certain is 1, or 100%.
 - All probabilities lie within this range.
- ✓ Two events are independent if the result of one event does not depend on the result of the other event.

What Should I Be Able to Do?

LESSON

- 7.1** **7.2** **1.** To celebrate his birthday, Justin and his friends played miniature golf. Here are their scores:
29, 33, 37, 24, 41, 38, 48, 26, 36, 33,
40, 29, 36, 22, 31, 38, 42, 35, 33
It was a par 36 course.

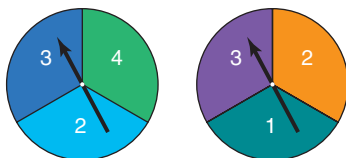
This means that a good golfer takes 36 strokes to complete the course.

- a) How many scores were under par? At par? Over par?
- b) Find the range of the scores.
- c) Calculate the mean, median, and mode scores.

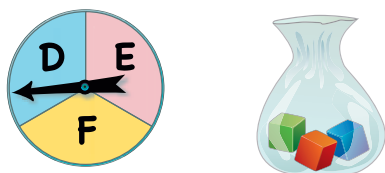
LESSON

- 7.2 2.** The median shoe size of eight 12-year-old boys is $6\frac{1}{2}$.
What might the shoe sizes be?
Explain your answer.
- 7.3 3.** Josephine recorded the hours she worked each week at her part-time job, for 10 weeks.
Here are the hours:
15, 12, 16, 10, 15, 15, 3, 18, 12, 10
- Calculate the mean, median, and mode hours.
 - Identify the outlier. How might you explain this value?
 - Calculate the mean, median, and mode hours without the outlier. How is each measure affected when the outlier is not included?
 - Should the outlier be used in reporting the average number of hours Josephine worked? Explain.
- 7.4 4.** The times, in minutes, that 14 students spent doing math homework over the weekend are:
27, 36, 48, 35, 8, 40, 41,
39, 74, 47, 44, 125, 37, 47
- Calculate the mean, median, and mode for the data.
 - What are the outliers? Justify your choice. Calculate the mean without the outliers. What do you notice? Explain.
 - Which average best describes the data? Explain.
 - Should the outliers be used in reporting the average? Explain.
- 5.** Annette's practice times for a downhill ski run, in seconds, are:
122, 137, 118, 119, 124, 118, 120, 118
- Find the mean, median, and mode times.
 - Which measure best represents the times? Explain.
 - What is the range?
 - What time must Annette get in her next run so the median is 120 s? Explain.
 - What time must Annette get in her next run so the mean is 121 s? Is this possible? Explain.
- 6.** Which average best describes a typical item in each data set? Explain.
- size of sandals sold in a shoe store
 - test scores of a Grade 7 class
 - salaries in a company with 1000 employees
 - salaries in a company with 6 employees
- 7.5 7.** Twenty cards are numbered from 1 to 20. The cards are shuffled. A card is drawn. Find the probability that the card has:
- an odd number
 - a multiple of 4
 - a prime number
 - a number greater than 30
 - a number divisible by 1
- Express each probability as many ways as you can.

- 7.6 8.** In a board game, players take turns to spin pointers on these spinners. The numbers the pointers land on are multiplied. The player moves that number of squares on the board.



- List the possible products.
 - What is the probability of each product in part a?
 - Which products are equally likely? Explain.
 - What is the probability of a product that is less than 10? Explain.
- 9.** A spinner has 3 equal sectors labelled D, E, and F. A bag contains 3 congruent cubes: 1 green, 1 red, and 1 blue



The pointer is spun and a cube is picked at random.

- Use a tree diagram to list the possible outcomes.
- What is the probability of:
 - spinning E?
 - picking a green cube?
 - spinning E and picking a green cube?
 - spinning D and picking a red cube?

- Record the results for 10 trials.
 - State the experimental probability for each event in question 9, part b.
 - How do the experimental and theoretical probabilities compare?
- Combine your results with those of 9 other students. You now have the results of 100 trials. Repeat part a.
- What happens to the experimental and theoretical probabilities of an event when the experiment is repeated hundreds of times?

- 11.** The student council runs a coin toss game during School Spirit week. All the profits go to charity. Each player pays 50¢ to play. A player tosses two coins and wins a prize if the coins match. Each prize is \$2.00. Can the student council expect to make a profit? Justify your answer.



Practice Test

- The data show the time, in seconds, for swimmers to swim a 400-m freestyle race.
208, 176, 265, 222, 333, 237, 225, 269, 303, 295, 238, 175, 257, 208, 271, 210
 - What is the mean time?
 - What is the mode time?
 - What is the range of the times?
 - What is the median time?
- A sports store carries women's ice skates, sizes 4 to 10. In a particular 4-h period, these skate sizes are sold:
4, 9, 8.5, 7.5, 6, 7, 6.5, 7, 7.5, 9, 8, 8, 18, 6.5, 8.5, 7, 5, 7, 9.5, 7
 - Calculate the mean, median, and mode sizes.
Which average best represents the data? Explain.
 - Identify the outlier. How can you explain this size?
 - Calculate the mean, median, and mode sizes without the outlier.
How is each average affected when the outlier is not included?
 - Should the outlier be used when the sales clerk reports the average skate size sold? Explain.
- Match each probability to an event listed below.
 - 0
 - $\frac{1}{2}$
 - 100%
 - 1:4
 - You roll an even number on a die labelled 1 to 6.
 - You pick an orange out of a bag of apples.
 - You draw a red counter from a bag that contains 3 yellow counters and 1 red counter.
 - You roll a number less than 7 on a die labelled 1 to 6.
- Two dice are rolled. Each die is labelled 1 to 6. The lesser number is subtracted from the greater number, to get the difference. For example: $5 - 3 = 2$
 - List the possible differences.
 - Express the probability of each difference as a ratio, fraction, and percent.
 - exactly 1
 - greater than 3
 - an odd number
 - Carry out this experiment 10 times. Record your results.
 - What is the experimental probability of each event in part b?
 - Compare the theoretical and experimental probabilities.
What do you notice?

Many board games involve rolling pairs of dice, labelled 1 to 6. Suppose you are on a particular square of a game board. You roll the dice. You add the numbers on the uppermost faces. How likely are you to roll 7? Investigate to find out.

Part 1

Work with a partner.
Copy and complete this table.
Show the sums when two dice are rolled.

When one die shows 1 and the other die shows 4, then the sum is 5.

Sum of Numbers on Two Dice						
Number on Die	1	2	3	4	5	6
1	2	3	4	5	6	7
2				6		
3						
4	5					
5						
6						

How many different outcomes are there?
In how many ways can the sum be 6? 9? 2? 12?
Why do you think a table was used instead of a tree diagram?
Find the theoretical probability for each outcome.

Part 2

Work in groups of four.

- Choose one of the following options:
 - Each of you rolls a pair of dice 25 times. Combine the results of your group.
 - Use a website your teacher gives you. Use the website to simulate rolling a pair of dice 100 times. Record the sum each time.

Calculate the experimental probability of each outcome after 100 rolls.





2. Repeat *Step 1* three more times, to get results for 200 rolls, 300 rolls, and 400 rolls.
Calculate the experimental probability of each outcome after each number of rolls.
Summarize your results for 400 rolls in a table.

Sum	Number of Times Sum Occurred	Experimental Probability	Theoretical Probability
2			
3			
4			
5			

Check List

Your work should show:

- ✓ all records and calculations in detail
- ✓ clear explanations of your results, with the correct use of language of data analysis
- ✓ tables to show outcomes and probabilities
- ✓ an explanation of who is more likely to win

Find the mode of the sum rolled.
You can use a software program to help.
What does this tell you?

Part 3

What happens to the experimental probabilities as the number of rolls increases?

How does the experimental probability of each outcome compare with the theoretical probability? Explain.

Part 4

To win a board game, you must land on "home."
Suppose you are 7 squares from "home."
Your opponent is 4 squares from "home."
Who is more likely to win on the next roll? Explain.
Why do you think 7 is a lucky number?

Reflect on Your Learning

Why do you think you are learning about data analysis?