UNIT 6

Equations

Many digital music clubs offer albums for subscribers to download.

These tables show the plans for downloading albums for two companies. Each plan includes 5 free album downloads per month.

What patterns do you see in the tables? Write a pattern rule for each pattern. Describe each plan.

Which is the more expensive plan for 8 additional albums? Assume the patterns continue. Will this company always be more expensive? How do you know?

What You'll Learn

- Demonstrate and use the preservation of equality.
- Explain the difference between an expression and an equation.
- Use models, pictures, and symbols to solve equations and verify the solutions.
- Solve equations using algebra.
- Make decisions about which method to use to solve an equation.
- Solve problems using related equations.

Why It's Important

- Using equations is an effective problem-solving tool.
- Using algebra to solve equations plays an important role in many careers. For example, urban planners use equations to investigate population growth.

Key Words

- systematic trial
- inspection

Company A

Number of Additional Albums Downloaded per Month	Total Cost ($)
0	10
1	18
2	26
3	34
4	42
5	50

Company B

Number of Additional Albums Downloaded per Month	Total Cost ($)
0	20
1	27
2	34
3	41
4	48
5	55

6.1 Solving Equations

Focus Solve equations by inspection and by systematic trial.

Look at the algebraic expressions and equations below.
Which ones are equations? Which ones are expressions?
How do you know?

$3n + 12$ $\quad$ $3n = 12$ $\quad$ $5x + 2$ $\quad$ $5x + 2 = 27$

Explore

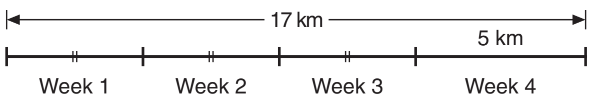
On the way home from school,
10 students got off the bus at the first stop.
There were then 16 students on the bus.
How many students were on the bus
when it left the school?
How many different ways can you solve the problem?

Reflect & Share

Discuss your strategies for finding the answer
with another pair of classmates.
Did you use an equation?
Did you use reasoning?
Did you draw a picture?
Justify your choice.

Connect

Janet walked a total of 17 km in February.
She walked the same number of kilometres
in each of the first 3 weeks.
Then she walked 5 km in the fourth week.
How many kilometres did Janet walk
in each of the first 3 weeks?



Let d represent the distance Janet walked, in kilometres, in each of the first 3 weeks.
So $3 \times d$, or $3d$, represents the total number of kilometres Janet walked in the first 3 weeks.
She walked 5 km in the fourth week, for a total of 17 km.
The equation is: $3d + 5 = 17$

Writing and solving equations is a useful strategy for solving problems.

When we use the equation to find the value of d, we *solve the equation*.

Here are 2 ways to solve this equation.

Method 1: By Systematic Trial

***Systematic trial* means choosing a value for the variable, then checking by substituting. Use the answer and reasoning to choose the next value to check.**

$3d + 5 = 17$
We choose a value for d and substitute.

Try $d = 2$.
$$\begin{aligned} 3d + 5 &= 3 \times 2 + 5 \\ &= 6 + 5 \\ &= 11 \end{aligned}$$
11 is too small, so choose a greater value for d.

Try $d = 5$.
$$\begin{aligned} 3d + 5 &= 3 \times 5 + 5 \\ &= 15 + 5 \\ &= 20 \end{aligned}$$
20 is too large, so choose a lesser value for d.

Try $d = 4$.
$$\begin{aligned} 3d + 5 &= 3 \times 4 + 5 \\ &= 12 + 5 \\ &= 17 \end{aligned}$$
This is correct.
Janet walked 4 km during each of the first 3 weeks of February.

Method 2: By Inspection

***By inspection* means finding the value for the variable by using these types of number facts: addition, subtraction, multiplication, and division**

$3d + 5 = 17$
We first find a number which, when added to 5, gives 17.
$$3d + 5 = 17$$
We know that $12 + 5 = 17$.
So, $3d = 12$
Then we find a number which, when multiplied by 3, has product 12.
We know that $3 \times 4 = 12$; so $d = 4$
Janet walked 4 km during each of the first 3 weeks of February.

We say that the value $d = 4$ makes the equation $3d + 5 = 17$ true.
Any other value of d, such as $d = 6$,
would not make the equation true.
The value $d = 4$ is the only solution to the equation.
That is, there is only one value of d that makes the equation true.

$3 \times 6 + 5$ does not equal 17.

Example

For each situation, write an equation.
Ben has a large collection of baseball caps.

a) Ben takes y caps from a group of 18 caps.
There are 12 caps left.
How many caps did Ben take away?
Solve the equation by inspection.

b) Ben put k caps in each of 6 piles.
There are 108 caps altogether.
How many caps did Ben put in each pile?
Solve the equation by systematic trial.

c) Ben shares n caps equally among 9 piles.
There are 6 caps in each pile.
How many caps did Ben have?
Solve the equation by inspection.

d) Ben combines p groups of 4 caps each into one large group.
He then takes away 7 caps. There are 49 caps left.
How many groups of 4 caps did Ben begin with?
Solve the equation by systematic trial.

A Solution

a) 18 subtract y equals 12.
$18 - y = 12$
Which number subtracted from 18 gives 12?
We know that $18 - 6 = 12$; so $y = 6$.
Ben took away 6 caps.

b) 6 times k equals 108.
$6k = 108$
Try $k = 15$. $\quad 6k = 6(15)$
$\quad\quad\quad\quad\quad\quad = 90$
90 is too small, so choose a greater value for k.

Recall: $6(15) = 6 \times 15$

Try $k = 20$. $6k = 6(20)$
$= 120$

120 is too large, so choose a lesser value for k.

Try $k = 17$. $6k = 6(17)$
$= 102$

102 is too small, but it is close to the value we want.

Try $k = 18$. $6k = 6(18)$
$= 108$

This is correct.

Ben put 18 caps in each pile.

c) n divided by 9 equals 6.

$n \div 9 = 6$, or $\frac{n}{9} = 6$

Which number divided by 9 gives 6?

We know that $54 \div 9 = 6$; so $n = 54$.

Ben had 54 caps.

d) 4 times p subtract 7 equals 49.

$4p - 7 = 49$

Since $4 \times 10 = 40$, we know we need to start with a value for p greater than 10.

Try $p = 12$. $4p - 7 = 4 \times 12 - 7$
$= 48 - 7$
$= 41$, which is too small

41 is 8 less than 49, so we need two more groups of 4.

Try $p = 14$. $4p - 7 = 4 \times 14 - 7$
$= 56 - 7$
$= 49$

This is correct.

Ben began with 14 groups of 4 caps each.

Practice

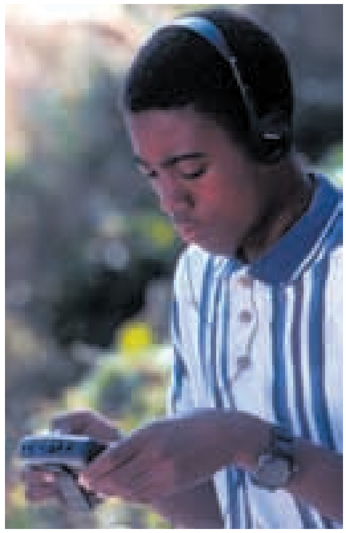
1. Look at the algebraic expressions and equations below.
Which are expressions? Equations?
How do you know?

a) $4w = 48$ **b)** $g - 11$ **c)** $3d + 5$

d) $\frac{x}{12} = 8$ **e)** $\frac{j - 5}{10}$ **f)** $6z + 1 = 67$

2. Solve each equation in question 1 by inspection or by systematic trial.
Explain why you chose the method you did.

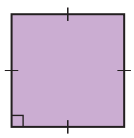
3. Shenker gives 10 CDs to his brother.
Shenker then has 35 CDs.
   a) Write an equation you can solve to find how many CDs Shenker had to begin with.
   b) Solve the equation.



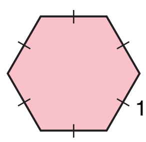
4. Write an equation for each sentence.
Solve each equation by inspection.
   a) Seven more than a number is 18.
   b) Six less than a number is 24.
   c) Five times a number is 45.
   d) A number divided by six is 7.
   e) Three more than four times a number is 19.

5. Write an equation you could use to solve each problem.
Solve each equation by systematic trial.
   a) Aiko bought 14 DVDs for $182.
   She paid the same amount for each DVD.
   How much did each DVD cost?
   b) Kihew collects beaded leather bracelets. She lost 14 of her bracelets.
   Kihew has 53 bracelets left.
   How many bracelets did she have to begin with?
   c) Manuel gets prize points for reading books.
   He needs 100 points to win a set of tangrams.
   Manuel has 56 points. When he reads 11 more books,
   he will have 100 points.
   How many points does Manuel get for each book he reads?

6. The perimeter of a square is 48 cm.
   a) Write an equation you can solve to find the side length of the square.
   b) Solve the equation.



7. The side length of a regular hexagon is 11 cm.
   a) Write an equation you can solve to find the perimeter of the hexagon.
   b) Solve the equation.



8. Use questions 6 and 7 as a guide.

a) Write your own problem about side length and perimeter of a figure.

b) Write an equation you can use to solve the problem.

c) Solve the equation.

9. **Assessment Focus** Eli has 130 key chains. He keeps 10 key chains for himself, then shares the rest equally among his friends.
Each friend then has 24 key chains.

a) Write an equation you can solve to find how many friends were given key chains.

b) Solve the equation by inspection, then by systematic trial. Which method was easier to use? Explain your choice.

10. Find the value of n that makes each equation true.

a) $3n = 27$ b) $2n + 3 = 27$ c) $2n - 3 = 27$ d) $\frac{n}{3} = 27$

11. **Take It Further** Write a problem that can be described by each equation. Solve each equation. Which equation was the most difficult to solve? Why do you think so?

a) $2x - 1 = 5$ b) $4y = 24$

c) $\frac{z}{38} = 57$ d) $5x + 5 = 30$

Math Link

Dr. Edward Doolittle, a Mohawk Indian, was the first Indigenous person in Canada to obtain a PhD in Mathematics. Dr. Doolittle has taught at the First Nations University in Saskatchewan, and he is currently an Assistant Professor of Mathematics at the University of Regina. One of Dr. Doolittle's goals is to show his students how much fun mathematics can be. In addition to his academic interests, Dr. Doolittle also writes and performs comedy sketches for radio.

Reflect

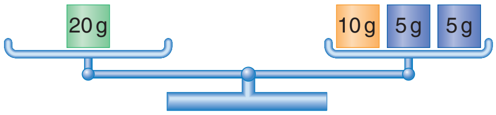
How do you decide whether to solve an equation by inspection or by systematic trial?
How might using a calculator affect your decision?
Give examples to illustrate your thinking.

6.2 Using a Model to Solve Equations

Focus Solve equations by using a balance-scales model and verifying the solution.

Sometimes, systematic trial and inspection are not the best ways to solve an equation.

Balance scales can be used to *model* an equation. When pans are balanced, the mass in one pan is equal to the mass in the other pan.

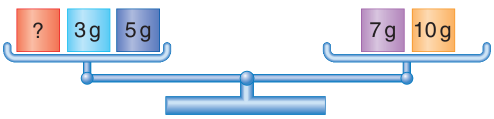


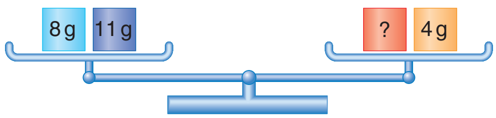
We can write an equation to describe the masses in grams.
$20 = 10 + 5 + 5$

Explore

Use balance scales if they are available.
Otherwise, draw diagrams.

Here are some balance scales.
Some masses are known. Other masses are unknown.





➤ The pans are balanced.
For each balance scales:
- Write an equation to represent the masses.
- Find the value of the unknown mass.

➤ Make up your own balance-scales problem.
Make sure the pans are balanced and one mass is unknown.
Solve your problem.

Reflect & Share

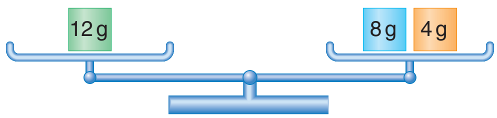
Trade problems with another pair of classmates.
Compare strategies for finding the value of the unknown mass.

Connect

We can use a balance-scales model to solve an equation.

When the two pans of scales are balanced,
we can adjust each pan of the scales in the same way,
and the two pans will still be balanced.

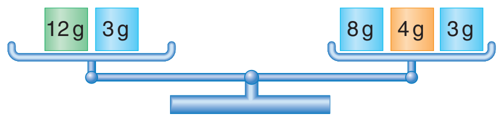
➤ Consider these balance scales:



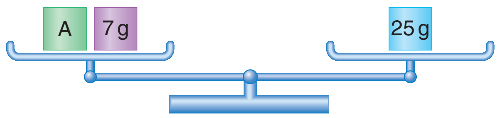
If we add 3 g to each pan,
the masses still balance.

$12\text{ g} + \mathbf{3\text{ g}} = 8\text{ g} + 4\text{ g} + \mathbf{3\text{ g}}$

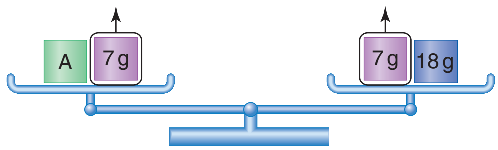
$15\text{ g} = 15\text{ g}$



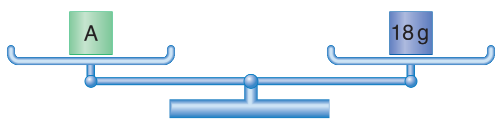
➤ Here is a balance-scales problem.
Mass A is an unknown mass.



If we remove 7 g from the left pan,
then Mass A is alone in that pan.
To keep the pans balanced,
we need to remove 7 g from the right pan too.
We replace 25 g with 7 g and 18 g;
then remove 7 g.



We are left with Mass A in the left pan
balancing 18 g in the right pan.
So, Mass A is 18 g.



We can *verify* the solution to this problem.
Replace Mass A with 18 g.
Then, in the left pan: $18\text{ g} + 7\text{ g} = 25\text{ g}$
And, in the right pan: 25 g
Since the masses are equal, the solution is correct.

To verify means to check the solution is correct.

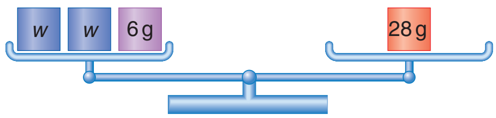
Example

A hockey team gets 2 points for a win,
1 point for a tie, and 0 points for a loss.
The Midland Tornadoes ended the season
with 28 points. They tied 6 games.
How many games did they win?
Write an equation you can use to solve the problem.
Use a model to solve the equation, then verify the solution.

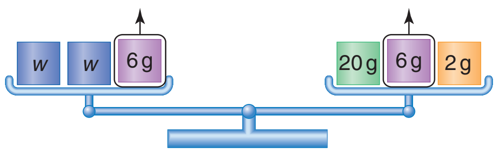
A Solution

Let w represent the number of games won by the Midland Tornadoes.
So, $2w$ represents the number of points earned from wins.
The team has 6 points from ties.
It has 28 points altogether.
So, the equation is: $2w + 6 = 28$

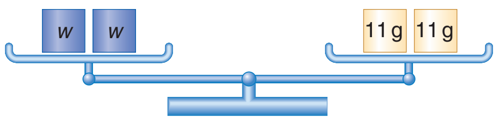
Use balance scales to model the equation.



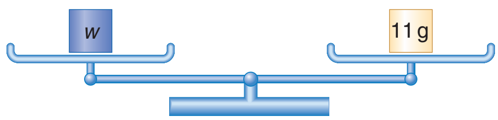
To get w on its own in one pan,
6 g has to be removed from the left pan.
So, select masses in the right pan
so that 6 g can be removed.
One way is to replace 28 g with 20 g + 6 g + 2 g.
Then, remove 6 g from each pan.



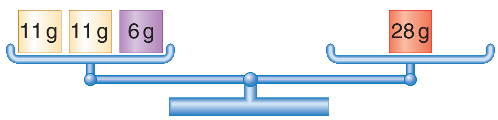
Two identical unknown masses
are left in the left pan.
20 g + 2 g = 22 g are left in the right pan.
Replace 22 g with two 11-g masses.



The two unknown masses balance
with two 11-g masses.
So, each unknown mass is 11 g.



The Midland Tornadoes won 11 games.
Verify the solution by replacing
each unknown mass with 11 g.
$11 + 11 + 6 = 28$, so the solution is correct.



The examples on pages 227 and 228 show these ways in which we preserve balance and equality:

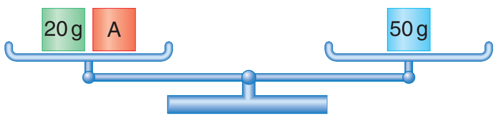
- We can *add* the same mass to each side.
- We can *subtract* the same mass from each side.
- We can *divide* each side into the same number of equal groups.

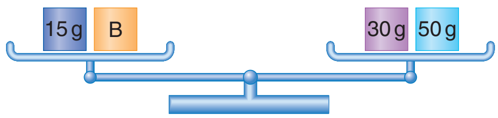
Later, we will show that:

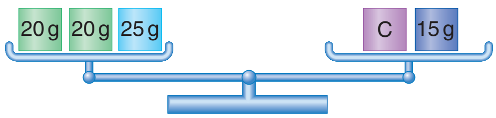
- We can *multiply* each side by the same number by placing equal groups on each side that match the group already there.

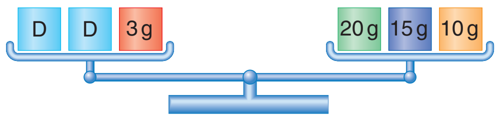
Practice

1. Find the value of the unknown mass on each balance scales. Sketch the steps you used.

a) 

b) 

c) 

d) 

2. a) Sketch balance scales to represent each equation.

b) Solve each equation. Verify the solution.

i) $x + 12 = 19$ ii) $x + 5 = 19$

iii) $4y = 12$ iv) $3m = 21$

v) $3k + 7 = 31$ vi) $2p + 12 = 54$

3. a) Write an equation for each sentence.

b) Solve each equation. Verify the solution.

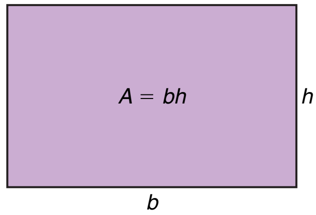
i) Five more than a number is 24.

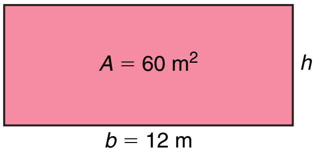
ii) Eight more than a number is 32.

iii) Three times a number is 42.

iv) Five more than two times a number is 37.

4. The area of a rectangle is $A = bh$,
where b is the base of the rectangle and h is its height.
Use this formula for each rectangle below.
Substitute for A and b to get an equation.
Solve the equation for h to find the height.
Show the steps you used to get the answers.



a)


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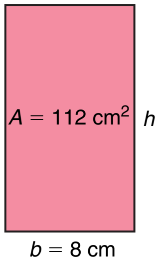

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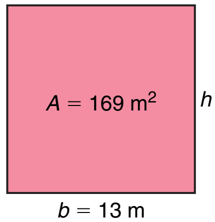


5. **Assessment Focus** Suppose the masses for balance scales are only available in multiples of 5 g.
   a) Sketch balance scales to represent this equation:
   $x + 35 = 60$
   b) Solve the equation. Verify the solution.
   Show your work.

6. **Take It Further**
   a) Write a problem that can be solved using this equation: $x + 4 = 16$
   b) How would your problem change if the equation were $x - 4 = 16$?
   c) Solve the equations in parts a and b.
   Show your steps.

7. **Take It Further** Write an equation you could use to solve this problem.
Replace the □ in the number 5□7 with a digit to make the number divisible by 9.

Reflect

Do you think you can always solve an equation using balance scales?
Justify your answer. Include an example.

6.3 Solving Equations Involving Integers

Focus Use algebra tiles and inspection to solve equations involving integers.

Recall that 1 red unit tile and 1 yellow unit tile combine to model 0.
These two unit tiles form a zero pair.

The yellow variable tile represents a variable, such as *x*.

Explore

You will need algebra tiles.
Tyler had some gumdrops and jellybeans.
He traded 5 gumdrops for 5 jellybeans.
Tyler then had 9 gumdrops and 9 jellybeans.
How many gumdrops did he have to begin with?

Let *g* represent the number of gumdrops Tyler began with.
Write an equation you can use to solve for *g*.
Use tiles to represent the equation.
Use the tiles to solve the equation. Sketch the tiles you used.

Reflect & Share

Compare your equation with that of another pair of classmates.
Share your strategies for solving the equation using tiles.
How did you use zero pairs in your solutions?
Work together to find how many jellybeans Tyler began with.
Discuss your strategies for finding out.

Connect

Michaela has a collection of old pennies.
She sells 3 pennies to another collector.
Michaela then has 10 pennies left.
How many pennies did she have before she made the sale?

Let p represent the number of pennies Michaela had before she made the sale.
The equation is: $p - 3 = 10$
One way to solve this equation is to use tiles.

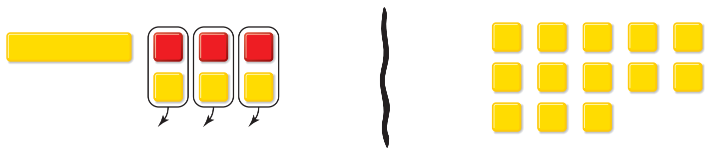
Draw a vertical line in the centre of the page.
It represents the equal sign in the equation.
We arrange tiles on each side of the line to represent an equation.
Recall that subtracting 3 is equivalent to adding -3.
So, we represent subtract 3 with 3 red unit tiles.

On the left side, put algebra tiles to represent $p - 3$.

On the right side, put algebra tiles to represent 10.

To isolate the variable tile, add 3 yellow unit tiles to make zero pairs.
Remove zero pairs.

Add 3 yellow unit tiles to this side, too, to preserve the equality.



The tiles show the solution is $p = 13$.
Michaela had 13 old pennies before she made the sale.

Recall from Unit 1 that we can verify the solution by replacing p with 13 yellow unit tiles.
Then:

becomes

Since there are now 10 yellow unit tiles on each side, the solution is correct.

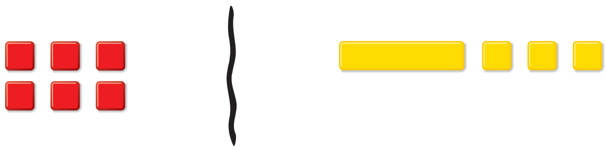
Example

At 10 a.m., it was cold outside.
By 2 p.m., the temperature had risen 3°C to −6°C.
What was the temperature at 10 a.m.?

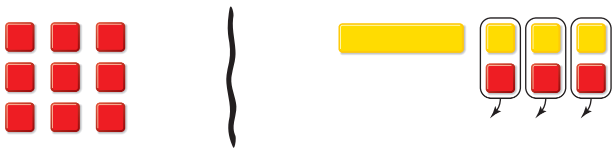
A Solution

Let t represent the temperature, in degrees Celsius, at 10 a.m.
After an increase of 3°C, the temperature was −6°C.
The equation is: $-6 = t + 3$

The variable in an equation can be on the left side or the right side.



Add 3 red unit tiles to each side. Remove zero pairs.



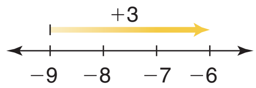
9 red unit tiles equals one variable tile.

We can verify the solution by replacing one yellow variable tile with 9 red unit tiles in the original equation.

The solution is $t = -9$.
At 10 a.m., the temperature was −9°C.

Another Solution

We can also solve equations involving integers by inspection.
To solve $-6 = t + 3$ by inspection:
We find a number which, when 3 is added to it, gives -6.
Think of moving 3 units to the right on a number line.
To arrive at -6, we would have to start at -9.
So $t = -9$.



Practice

1. Use tiles to solve each equation.
   Sketch the tiles you used.
   a) $x + 4 = 8$ b) $3 + x = 10$ c) $12 = x + 2$
   d) $x - 4 = 8$ e) $10 = x - 3$ f) $12 = x - 2$

2. Solve by inspection. Show your work.
   a) $9 = n - 4$ b) $x + 6 = 8$ c) $2 = p - 5$
   d) $x - 4 = -9$ e) $-8 = s + 6$ f) $x - 5 = -2$

3. Four less than a number is 13.
   Let x represent the number.
   Then, an equation is: $x - 4 = 13$
   Solve the equation. What is the number?

4. Jody had some friends over to watch movies.
   Six of her friends left after the first movie.
   Five friends stayed to watch a second movie.
   Write an equation you can use to find how many of Jody's friends watched the first movie.
   Solve the equation. Verify the solution.

5. Overnight, the temperature dropped 8°C to −3°C.
   a) Write an equation you can solve to find the original temperature.
   b) Use tiles to solve the equation.
      Sketch the tiles you used.

6. **Assessment Focus** Solve each equation using tiles, and by inspection.
Verify each solution. Show your work.

a) $x + 6 = 13$ b) $n - 6 = 13$

7. At the Jungle Safari mini-golf course, par on each hole is 5.
A score of -1 means a player took 4 strokes to reach the hole.
A score of $+2$ means a player took 7 strokes to reach the hole.
Write an equation you can use to solve each problem below.
Solve the equation. Show your work.

> **Par is the number of strokes a good golfer should take to reach the hole.**

a) On the seventh hole, Andy scored $+2$.
His overall score was then $+4$.
What was Andy's overall score after six holes?

b) On the thirteenth hole, Bethany scored -2.
Her overall score was then $+1$.
What was Bethany's overall score after twelve holes?

c) On the eighteenth hole, Koora reached the hole in one stroke.
His overall score was then -2.
What was Koora's overall score after seventeen holes?

8. **Take It Further** Consider equations of the form $x + a = b$, where a and b are integers. Make up a problem that can be solved by an equation of this form in which:

a) Both a and b are positive.
b) Both a and b are negative.
c) a is positive and b is negative.
d) a is negative and b is positive.

Solve each equation.
Explain the method you used each time.

Reflect

How did your knowledge of adding and subtracting integers help you in this lesson?

Mid-Unit Review

LESSON

6.1

1. Jaclyn went on a 4-day hiking trip.
   a) For each problem, write an equation you can solve by inspection.
      i) Jaclyn hiked 5 km the first day. After two days she had hiked 12 km. How far did she hike on the second day?
      ii) Jaclyn hiked a total of 12 km on the third and fourth days. She hiked the same distance each day. How far did she hike on each of these two days?
   b) For each problem, write an equation you can solve by systematic trial.
      i) Jaclyn counted squirrels. During the first three days, she counted a total of 67 squirrels. After four days, her total was 92 squirrels. How many squirrels did she see on the fourth day?
      ii) Jaclyn drank the same volume of water on each of the first three days. On the fourth day, she drank 8 cups of water. She drank 29 cups of water over the four days. How many cups of water did she drink on each of the first three days?

6.2

2. a) Write an equation for each sentence.
   b) Sketch balance scales to represent each equation.
   c) Solve each equation. Verify the solution.
      i) Nine more than a number is 17.
      ii) Three times a number is 21.
      iii) Seven more than two times a number is 19.

3. Andre's age is 14 more than twice Bill's age. Andre is 40 years old. How old is Bill? Write an equation you can use to solve this problem. Solve the equation using a balance-scales model.

6.3

4. a) Write an equation you can use to solve each problem.
   b) Use tiles to solve each equation. Sketch the tiles you used. Verify each solution.
   c) Solve each equation by inspection.
      i) Eight years ago, Susanna was 7 years old. How old is she now?
      ii) The temperature dropped 6°C to −4°C. What was the original temperature?
      iii) Hannah borrowed money. She paid back \$7. Hannah still owes \$5. How much money did she borrow?

6.4 Solving Equations Using Algebra

Focus Solve a problem by solving an equation algebraically.

Explore

Solve this problem.
My mother's age is 4 more than 2 times my brother's age.
My mother is 46 years old. How old is my brother?

Reflect & Share

Discuss the strategies you used for finding the brother's age with those of another pair of classmates.
Did you draw a picture? Did you use tiles? Did you use an equation?
If you did not use an equation, how could you represent this problem with an equation?

Connect

Solving an equation *using algebra* is often the quickest way to find a solution, especially if the equation involves large numbers.

Recall that when we solve an equation,
we find the value of the variable that makes the equation true.
That is, we find the value of the variable which,
when substituted into the equation,
makes the left side of the equation equal to the right side.

When we solve an equation using algebra, remember the balance-scales model.
To preserve the equality, always perform the same operation on both sides of the equation.

Whatever you do to one side of an equation, you do to the other side too.

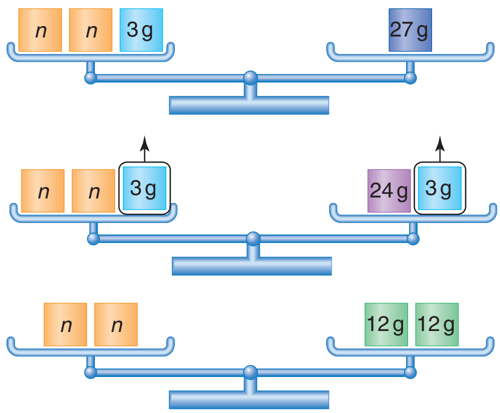
Example

Three more than two times a number is 27. What is the number?

a) Write an equation to represent this problem.
b) Solve the equation. Show the steps.
c) Verify the solution.

A Solution

a) Let n represent the number.
Then two times the number is: $2n$
And, three more than two times the number is: $2n + 3$
The equation is: $2n + 3 = 27$



b) $2n + 3 = 27$
To isolate $2n$, subtract 3 from each side.

$$2n + 3 - 3 = 27 - 3$$
$$2n = 24$$

Divide each side by 2.

$$\frac{2n}{2} = \frac{24}{2}$$
$$n = 12$$

c) To verify the solution, substitute $n = 12$ into $2n + 3 = 27$.

Left side $= 2n + 3$ Right side $= 27$
$= 2(12) + 3$
$= 24 + 3$
$= 27$

Since the left side equals the right side, $n = 12$ is correct.
The number is 12.

We could also verify by following the instructions in the *Example*.
Start with 12.
Multiply by 2: 24
Add 3: 27

Practice

Solve each equation using algebra.

1. Solve each equation. Verify the solution.
a) $x - 27 = 35$ **b)** $11x = 132$ **c)** $4x + 7 = 75$

2. Write, then solve, an equation to find each number. Verify the solution.
a) Nineteen more than a number is 42.
b) Ten more than three times a number is 25.
c) Fifteen more than four times a number is 63.

3. Five years after Jari's age now doubles, he will be 27. How old is Jari now?
a) Write an equation you can use to solve the problem.
b) Solve the equation. Show the steps. How old is Jari?
c) Verify the solution.

4. Jenny baby-sat on Saturday for \$6/h. She was given a \$3 bonus. How many hours did Jenny baby-sit if she was paid \$33?

a) Write an equation you can use to solve the problem.

b) Solve the equation. How many hours did Jenny baby-sit?

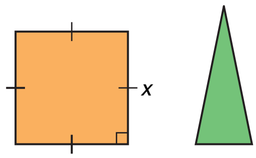
c) Verify the solution.

5. In x weeks and 4 days, the movie *Math-Man IV* will be released. The movie will be released in 25 days. Find the value of x.

a) Write an equation you can use to solve the problem.

b) Solve the equation. Verify the solution.

6. Look at the square and triangle on the right.
The sum of their perimeters is 56 cm.
The perimeter of the triangle is 24 cm.
What is the side length of the square?



a) Write an equation you can use to find the side length of the square.

b) Solve the equation. Verify the solution.

7. **Assessment Focus** Sunita has \$72 in her savings account. Each week she saves \$24. When will Sunita have a total savings of \$288?

a) Write an equation you can use to solve the problem.

b) Solve the equation. Show the steps.
When will Sunita have \$288 in her savings account?

c) How can you check the answer?

8. **Take It Further** Use the information on the sign to the right.

a) Write a problem that can be solved using an equation.

b) Write the equation, then solve the problem.

c) Show how you could solve the problem without writing an equation.

9. **Take It Further** The nth term of a number pattern is $9n + 1$. What is the term number for each term value?

a) 154 b) 118 c) 244

Reflect

What advice would you give someone who is having difficulty solving equations using algebra?

6.5 Using Different Methods to Solve Equations

Focus Decide which method to use to solve an equation.

Recall the methods you have used to solve an equation:
- using algebra tiles
- by inspection
- by systematic trial
- using a balance-scales model
- using algebra

Explore

Lila, Meeka, and Noel are playing darts.
Each player throws 3 darts at the board.
A player's score is the sum of the numbers in the areas the darts land.
This picture shows a score of:
$8 + 20 + 2 = 30$

Write an equation for each problem.
Solve the equation using a method of your choice.

➤ Lila's first two darts scored a total of 12 points.
Lila scored 20 points in the round.
How many points did she score with her third dart?

➤ All three of Meeka's darts landed in the same area.
She scored 63 points. In which area did all her darts land?

➤ Noel's first two darts landed in the same area.
Her third dart was a bull's-eye, scoring 50 points.
She scored a total of 72 points.
In which area did her first two darts land?

Reflect & Share

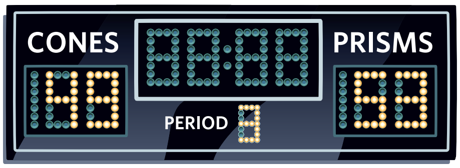
Compare your equations with those of another pair of classmates.
Explain why you chose the method you did to solve them.
Use a different method to solve one of the equations.
Did this method work better for you? Why do you think so?

Connect

We can use any method to solve an equation, as long as the steps we take make sense, and the correct solution is found.

You can always check if the solution is correct by substituting the solution into the original equation.

Example



In a basketball game between the Central City Cones and the Park Town Prisms, the lead changed sides many times.
Write, then solve, an equation to solve each problem.

a) Early in the game, the Cones had one-half as many points as the Prisms.
The Cones had 8 points. How many points did the Prisms have?

b) Near the end of the first half, the Cones were 12 points ahead of the Prisms.
The Prisms had 39 points. How many points did the Cones have?

c) The Prisms scored 32 points in the fourth quarter.
Twenty of these points were scored by foul shots and field goals.
The rest of the points were scored by 3-point shots.
How many 3-point shots did the Prisms make in the fourth quarter?

A Solution

a) Let p represent the number of points the Prisms had.
The Cones had one-half as many: $\frac{p}{2}$
The Cones had 8 points.
The equation is: $\frac{p}{2} = 8$
Solve using algebra.
Multiply each side by 2.

$$\frac{p}{2} \times 2 = 8 \times 2$$
$$\frac{2p}{2} = 16$$
$$p = 16$$

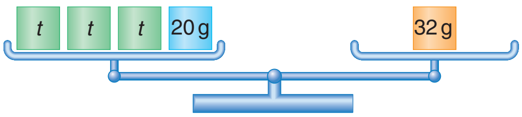
The Prisms had 16 points.

Another Strategy
We could use inspection to solve this equation.

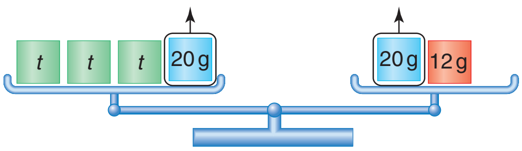
b) Let d represent the number of points the Cones had.
The Prisms had 12 fewer points: $d - 12$
An equation is: $d - 12 = 39$
Solve using systematic trial. We choose a value for d and substitute.
Try $d = 50$. $\quad d - 12 = 50 - 12$
$\quad\quad = 38$
38 is close. Choose a greater value for d.
Try $d = 51$. $\quad d - 12 = 51 - 12$
$\quad\quad = 39$
This is correct. The Cones had 51 points.

Another Strategy
We could use algebra to solve this equation.

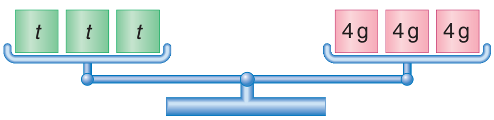
c) Let t represent the number of 3-point shots the Prisms made in the fourth quarter.
So, $3t$ represents the number of points scored by 3-point shots.
The equation is: $3t + 20 = 32$
Use a balance-scales model.



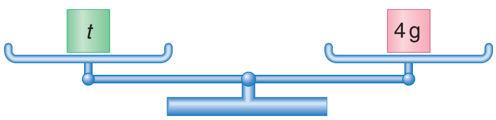
To isolate t, 20 g has to be removed from the left pan.
So, replace 32 g in the right pan with masses of 20 g and 12 g,
since 20 g + 12 g = 32 g. Then, remove 20 g from each pan.



Three identical unknown masses remain in the left pan.
12 g remain in the right pan. Replace 12 g with three 4-g masses.



The three unknown masses balance with three 4-g masses.
So, each unknown mass is 4 g.



The Prisms made four 3-point shots in the fourth quarter.

In *Example,* part c, we can solve the equation using algebra to check.

$$3t + 20 = 32$$

$$3t + 20 - 20 = 32 - 20 \qquad \text{To isolate } 3t\text{, subtract 20 from each side.}$$

$$3t = 12$$

$$\frac{3t}{3} = \frac{12}{3} \qquad \text{To isolate } t\text{, divide each side by 3.}$$

$$t = 4$$

The algebraic solution and the balance-scales solution are the same.
It was much quicker to solve the equation using algebra.

Practice

Use algebra, systematic trial, inspection, algebra tiles, or a balance-scales model to solve each equation.

1. Use algebra to solve each equation. Verify each solution.

a) $\frac{x}{2} = 4$ **b)** $\frac{x}{3} = 7$ **c)** $\frac{x}{4} = 16$ **d)** $\frac{x}{5} = 10$

2. Which method would you choose to solve each equation?
Explain your choice.
Solve each equation using the method of your choice.

a) $x + 5 = 12$ **b)** $x - 5 = 12$ **c)** $\frac{x}{6} = 9$ **d)** $x + 4 = -9$

e) $4x = 36$ **f)** $16x = 112$ **g)** $4x + 2 = 30$ **h)** $8x + 17 = 105$

3. George and Mary collect friendship beads.
George gave Mary 7 beads.
Mary then had 21 beads.
How many friendship beads did Mary have to start with?

a) Write, then solve, an equation you can use to solve this problem.

b) Verify the solution.

4. Jerome baked some cookies.
He shared them among his eight friends.
Each friend had 4 cookies.
Write, then solve, an equation to find how many cookies Jerome baked.

5. Which method do you prefer to use to solve an equation?
Explain. Give an example.

6. **Assessment Focus** Carla has 20 songs downloaded to her MP3 player.
Each month she downloads 8 additional songs.
After how many months will Carla have a total of 92 songs?
   a) Use an equation to solve the problem.
   b) Which method did you choose to solve the equation?
   Explain why you chose this method.

7. Write, then solve, an equation to answer each question.
Verify the solution.
Sheng sorted 37 cans.
   a) He divided the cans into 4 equal groups.
   He had 5 cans left over.
   How many cans were in each group?
   b) He divided the cans into 9 equal groups.
   He had 10 cans left over.
   How many cans were in each group?

8. Write, then solve, an equation to answer each question.
Verify the solution.
At Pascal's Pet Store, a 5-kg bag of dog food costs \$10.
The 10-kg bag costs \$15.
   a) Pascal sold \$85 worth of dog food.
   He sold four 5-kg bags. How many 10-kg bags did he sell?
   b) Pascal sold \$140 worth of dog food.
   He sold six 10-kg bags. How many 5-kg bags did he sell?

9. **Take It Further** Refer to the dart problem in *Explore*, page 240.
   a) Write two more problems using the given information.
   For each problem, write an equation you can use
   to solve your problem. Solve the equation.
   Use a different method for each equation.
   b) How could a player score 35 points with 3 darts?
   Find as many different ways as you can.

Reflect

Talk to a partner.
Tell how you choose the method you use to solve an equation.

Equation Baseball

Each game card is marked with a circled number to indicate how many bases you move for a correct answer. Each time a player crosses home plate on the way around the board, one run is awarded.

YOU WILL NEED

A set of Equation Baseball cards; one game board; a die; different coloured game pieces; algebra tiles; paper; pencils

NUMBER OF PLAYERS

4

GOAL OF THE GAME

To get the most runs

HOW TO PLAY THE GAME:

1. Shuffle the equation cards.
   Place them face down in the middle of the game board.
   Each player places a game piece on home plate.

2. Each player rolls the die. The player with the greatest number goes first. Play moves in a clockwise direction.

3. The first player turns over the top card for everyone to see.
   She solves the equation using a method of her choice.
   The other players check the answer.
   If the answer is correct, she moves the number of bases indicated by the circled number on the card and places the card in the discard pile. If the answer is incorrect, the card is placed in the discard pile.

4. The next player has a turn.

5. The player with the most runs when all cards have been used wins.

Decoding Word Problems

A word problem is a math question that has a story. Word problems often put math into real-world contexts.

The ability to read and understand word problems helps you connect math to the real world and solve more complicated problems.

Work with a partner. Compare these two questions.

Question 1	Question 2
Taylor has three more apples than Judy. Judy has six apples. How many apples does Taylor have?	$6 + 3$

List three reasons why the first question is more difficult than the second question.

A word problem can be challenging because it may not be obvious which math operations are needed ($+$, $-$, $\times$, $\div$) to solve it.

Reading and Writing in Math

Work with a partner.
Solve each word problem below.

1. Julio has 36 photos of his favourite singing star.
   He wants to arrange the photos in groups that have equal numbers of rows and columns.
   How many different arrangements can Julio make?
   Show your work.

2. A rectangular garden is 100 m long and 44 m wide.
   A fence encloses the garden.
   The fence posts are 2 m apart.
   Predict how many posts are needed.

3. A digital clock shows this time.
   Seven minutes past 7 is a palindromic time.
   List all the palindromic times between noon and midnight.

What's the Question?

The *key words* in a word problem are the words that tell you what to do.

Here are some common key words:

solve	explain
describe	predict
estimate	simplify
show your work	graph
find	list
compare	

Work with a partner. Discuss each question:
- What does each key word ask you to do?
- Which key words require an exact answer?
- Which key words tell you to show your thinking?

Unit Review

What Do I Need to Know?

☑ We can solve equations:
- by inspection
- by systematic trial
- using a balance-scales model
- using algebra tiles
- using algebra

☑ To keep the balance of an equation, what you do to one side you must also do to the other side. This is called preserving equality.

What Should I Be Able to Do?

LESSON

6.1

1. Jan collects foreign stamps.
Her friend gives her 8 stamps.
Jan then has 21 stamps.
How many stamps did Jan have to start with? Let x represent the number of stamps.
Then, an equation is: $8 + x = 21$
Solve the equation.
Answer the question.

2. Write an equation for each sentence.
Solve each equation by inspection.

a) Five more than a number is 22.
b) Seven less than a number is 31.
c) Six times a number is 54.
d) A number divided by eight is 9.
e) Nine more than three times a number is 24.

3. Write an equation you can use to solve each problem. Solve each equation by systematic trial.

a) Ned spent \$36 on a new shuttlecock racquet.
He then had \$45 left.
How much money did Ned have before he bought the racquet?

b) Laurie sold 13 books for \$208.
All books had the same price.
What was the price of each book?

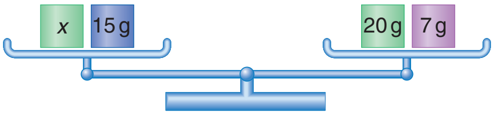
c) Maurice sorts some dominoes.
He divides them into 15 groups, with 17 dominoes in each group.
How many dominoes does Maurice sort?

LESSON

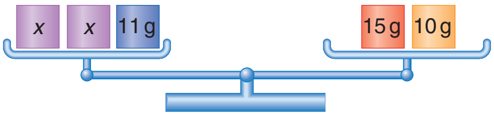
6.2

4. Write the equation that is represented by each balance scales. Solve the equation. Sketch the steps.

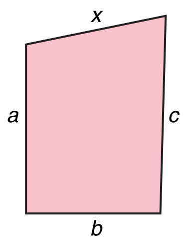
a)



b)



5. Look at the polygon below.



Recall that perimeter is the distance around.

Find the value of x when:

a) the perimeter of the polygon is 21 cm and $a = 5$ cm, $b = 3$ cm, and $c = 7$ cm

b) the perimeter of the polygon is 60 cm and $a = 15$ cm, $b = 11$ cm, and $c = 18$ cm

6. Jerry makes some photocopies. He pays 25¢ for a copy-card, plus 8¢ for each copy he makes. Jerry paid a total of 81¢. How many photocopies did Jerry make?

a) Write, then solve, an equation you can use to solve this problem. Show the steps.

b) Verify the solution.

6.3

7. Solve each equation using tiles, and by inspection. Verify each solution.

a) $x + 6 = 9$ b) $n + 9 = 6$

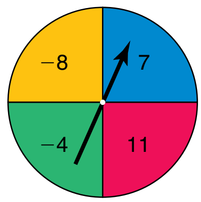
c) $w - 6 = 9$ d) $x - 9 = 6$

8. Adriano thinks of two numbers. When he adds 5 to the first number, the sum is -7. When he subtracts 5 from the second number, the difference is $+7$. What are the two numbers?

a) Write 2 equations you can use to solve this problem.

b) Use algebra tiles to solve the equations. Sketch the tiles you used.

9. Max spins the pointer on this spinner. He adds the number the pointer lands on to his previous total each time.



a) Write an equation you can use to solve each problem below.

b) Solve the equation using algebra tiles.

c) Verify each solution.

i) Max gets -8 on his first spin. After his next spin, his total is $+3$. Which number did he get on his second spin?

ii) After 3 spins, Max has a total of -1. Which number did he get on his third spin?

LESSON

6.4

10. Sara collects 56 leaves for a science project. She collects the same number of each of 7 different types of leaves. How many of each type did Sara collect?

a) Write an equation you can use to solve the problem.

b) Solve the equation. Verify the solution.

11. Serena walks 400 m from home to school. Serena is 140 m from school. How far is Serena from home?

a) Write an equation you can use to solve the problem.

b) Solve the equation using algebra. Verify the solution.

12. The Grade 7 classes sold pins to raise money for charity. They raised $228. Each pin sold for $4. How many pins did they sell?

a) Write an equation you can use to solve the problem.

b) Solve the equation using algebra. Verify the solution.

13. Write a problem that can be described by each equation below. Solve each equation using algebra. Explain the meaning of each answer.

a) $x + 15 = 34$ **b)** $7x = 49$

c) $\frac{x}{5} = 9$ **d)** $4x + 5 = 37$

6.5

14. Solve each equation using a method of your choice. Explain why you chose each method.

a) $x + 12 = 24$ **b)** $x + 7 = -3$

c) $x - 18 = -15$ **d)** $4x = 28$

e) $\frac{x}{11} = 9$ **f)** $5x + 8 = 73$

15. Jaya has 25 hockey cards. She has one more than 3 times the number of cards her brother has. Write, then solve, an equation to find how many cards he has.

16. The school's sports teams held a banquet. The teams were charged $125 for the rental of the hall, plus $12 for each meal served. The total bill was $545. How many people attended the banquet?

a) Write an equation you could use to solve the problem.

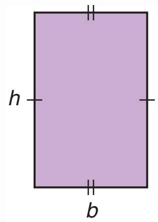
b) Solve your equation.

c) Verify the solution.

Practice Test

1. Solve each equation using a method of your choice.
Explain your steps clearly.
a) $x - 9 = -7$ **b)** $12p = 168$ **c)** $\frac{c}{7} = 9$ **d)** $7q + 11 = 102$

2. The area of a rectangle is $A = bh$,
where b is the base of the rectangle and h is its height.
The perimeter of a rectangle is $P = 2b + 2h$.
- Write an equation you can use to solve each problem below.
- Solve the equation. Verify the solution.
  a) What is the height of the rectangle when $b = 4$ cm and $A = 44$ cm^2?
  b) What is the base of the rectangle when $h = 16$ cm and $P = 50$ cm?



3. The formula $s = \frac{d}{t}$ relates average speed, s, distance, d, and time, t.
Brad took part in a mini-marathon race.
a) Brad jogged at an average speed of 5 km/h for 2 h.
How far did he jog?
b) Brad then rode his bike at an average speed of 16 km/h for 3 h.
How far did he ride his bike?
c) What distance was the race?
Show how you used the formula to solve this problem.

4. Wapeka saves pennies. She has 12¢ in her jar at the start.
Wapeka starts on January 1st. She saves 5¢ every day.
- Write an equation you can use to solve each problem.
- Solve the equation. Verify the solution.
  a) By which day had Wapeka saved a total of 47¢?
  b) By which day had Wapeka saved a total of $1.07?

5. Anoki is holding a skating party.
The rental of the ice is $75, plus $3 per skater.
a) Write an expression for the cost in dollars for 25 skaters.
b) Suppose Anoki has a budget of $204. Write an equation
you can solve to find how many people can skate.
Solve the equation.

Unit Problem Choosing a Digital Music Club

Suppose your older sister has bought an MP3 player.
She wants to download songs to play on it.
She asks for your help to find the best digital music club.

Part 1

1. Here are three plans from digital music clubs.
Each plan includes 10 free downloaded songs per month.

Songs4U: $20 per month, plus $3 per additional song
YourHits: $30 per month, plus $2 per additional song
Tops: $40 per month, plus $1 per additional song

Copy and complete this table.

Number of Additional Songs per Month	3	6	9	12	15
Cost of Songs4U ($)					
Cost of YourHits ($)					
Cost of Tops ($)					

What patterns do you see in the table?

2. Which club would you recommend if your sister plans to download 3 additional songs per month? 9 additional songs? 15 additional songs? Explain your choice each time.

Part 2

3. a) For each plan, write an expression for the monthly cost of n additional songs.

b) Use each expression to find the total monthly cost for 10 additional songs for each club.

c) Suppose your sister can afford to spend $80 a month on downloading songs. Write an equation you can solve to find how many additional songs she can afford with each plan.

d) Solve each equation. Explain what each solution means.

Part 3

Write a paragraph to explain what decisions you made about choosing the best digital music club.

Check List

Your work should show:

- ✓ the completed table
- ✓ the expressions and equations you wrote, and how you used them to solve the problems
- ✓ detailed, accurate calculations
- ✓ clear explanations of your solutions

Reflect on Your Learning

You have learned different methods to solve an equation.
Which method do you prefer? Why?
Which method do you find most difficult?
What is it that makes this method so difficult?

Units 1–6 Cumulative Review

UNIT

1

1. Use the divisibility rules to find the factors of each number.
   a) 120 b) 84 c) 216

2. Grace collects autographs of sports celebrities. She collected 7 autographs at the BC Open Golf Tournament. She then had 19 autographs. How many autographs did she have before the BC Open?
   a) Write an equation you can solve to find how many autographs Grace had before the BC Open.
   b) Solve the equation.
   c) Verify the solution.

2

3. Write the integer suggested by each situation below. Draw yellow or red tiles to model each integer.
   a) You walk down 8 stairs.
   b) You withdraw $10 from the bank.
   c) The temperature rises 9°C.

4. Use tiles to subtract.
   a) $(-9) - (-3)$ b) $(+9) - (-3)$
   c) $(+9) - (+3)$ d) $(-9) - (+3)$

3

5. Find a number between each pair of numbers.
   a) 1.6, 1.7 b) $\frac{6}{11}, \frac{7}{11}$
   c) $2\frac{1}{7}, \frac{16}{7}$ d) $2.7, 2\frac{4}{5}$

6. Find the area of a rectangular vegetable plot with base 10.8 m and height 5.2 m.

7. The regular price of a scooter is $89.99. The scooter is on sale for 20% off.
   a) What is the sale price of the scooter?
   b) There is 14% sales tax. What would a person pay for the scooter?

4

8. a) How many radii does a circle have?
   b) How many diameters does a circle have?

9. A DVD has diameter 12 cm.
   a) Calculate the circumference of the DVD. Give your answer to two decimal places.
   b) Estimate to check if your answer is reasonable.

10. Which has the greatest area? The least area?
    a) a rectangle with base 10 cm and height 5 cm
    b) a parallelogram with base 7 cm and height 8 cm
    c) a square with side length 7 cm

11. Zacharie has 50 m of plastic edging. He uses all the edging to enclose a circular garden. Find:
    a) the circumference of the garden
    b) the radius of the garden
    c) the area of the garden

UNIT

12. Adele recorded the hair colours of all Grades 7 and 8 students in her school.

Hair Colour	Students
Black	60
Brown	20
Blonde	30
Red	10

a) Find the total number of students.
b) Write the number of students with each hair colour as a fraction of the total number of students.
c) Write each fraction as a percent.
d) Draw a circle graph to represent the data.

5

13. A cookie recipe calls for $\frac{3}{8}$ cup of brown sugar and $\frac{1}{3}$ cup of white sugar. How much sugar is needed altogether? Show your work.

14. Estimate, then add or subtract.

a) $\frac{3}{5} + \frac{1}{6}$ b) $\frac{5}{6} - \frac{5}{12}$
c) $\frac{2}{3} - \frac{1}{8}$ d) $\frac{1}{4} + \frac{2}{9}$

15. The Boudreau family started a trip with the gas gauge reading $\frac{3}{4}$ full. At the end of the trip, the gauge read $\frac{1}{8}$ full. What fraction of a tank of gas was used?

16. Add or subtract.

a) $5\frac{1}{6} + 3\frac{3}{4}$ b) $1\frac{3}{10} - \frac{2}{3}$
c) $1\frac{3}{5} + 3\frac{2}{3}$ d) $2\frac{5}{6} - 1\frac{5}{8}$

6

17. a) Sketch balance scales to represent each equation.
b) Solve each equation. Verify the solution.
i) $s + 9 = 14$ ii) $s + 5 = 14$
iii) $3s = 27$ iv) $3s + 5 = 23$

18. Use tiles to solve each equation. Sketch the tiles you used.

a) $x + 5 = 11$ b) $13 = x - 4$

19. Juan works as a counsellor at a summer camp. He is paid \$7/h. He was given a \$5 bonus for organizing the scavenger hunt. How many hours did Juan work if he was paid \$250?

a) Write an equation you can use to solve the problem.
b) Solve the equation. How many hours did Juan work?

20. In a game of cards, black cards are worth $+1$ point each, and red cards are worth -1 point each. Four players are each dealt 13 cards per round, their scores are recorded, then the cards are shuffled. Write an equation you can use to solve each problem below. Solve the equation.

a) In Round Two, Shin was dealt 8 black cards and 5 red cards. His overall score was then $+10$. What was Shin's score after Round One?
b) In Round Two, Lucia was dealt 6 black cards and 7 red cards. Her overall score was then -4. What was Lucia's score after Round One?